A NUMERICAL EXPERIMENT ON 2-D TURBULENCE ON A ROTATING SPHERE

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ABSTRACT. This is an extended abstract on the numerical study of two-dimensional turbulence on a rotating sphere (Yoden and Yamada 1992). Temporal variations of the total kinetic energy, the total enstrophy and the enstrophy dissipation rate are found to be influenced by both the spherical geometry and the rotation rate. Morphology of streamfunction and vorticity fields is investigated for several rotation rates. In non-rotational cases, isolated coherent vortices emerge in the course of time development as in the planar two-dimensional turbulence. As the rotation rate increases, however, the evolution of the flow field changes drastically and there appears an easterly circumpolar vortex in high latitudes. The flow field is then anisotropic in all the latitudes and elongated in the longitudinal direction. Evolution of the flow field is characterized by Rossby wave motions.

1. Introduction

A series of numerical experiments on the decaying two-dimensional turbulence are performed for a nondivergent barotropic fluid on a rotating sphere by using a high-resolution spectral model. Basic dynamical properties of the turbulence are investigated in spherical geometry, and the effects of rotation are clarified in a series of experiments with several rotation rates. Evolution of the flow field is investigated as well as that of statistical quantities including the energy spectrum.

2. Model and experiment

Two-dimensional nondivergent flow on a rotating sphere is governed by a vorticity equation:

\[ \frac{\partial \omega}{\partial t} + \frac{1}{a^2} J(\psi, \omega) + \frac{2\Omega}{a^2} \frac{\partial \psi}{\partial \lambda} = - \nu_4 \Delta^2 \omega \]  

(1)

where \( \psi(\lambda, \mu, t) \) is a streamfunction field, \( \omega(\lambda, \mu, t) \) vorticity (= \( \Delta \psi \)), \( \lambda \) longitude, \( \mu \) sine latitude, \( t \) time, \( J(\psi, \omega) \) horizontal Jacobian, \( \Delta \) horizontal Laplacian, \( a \) radius of the sphere, \( \Omega \) rotation rate of the sphere, and \( \nu_4 \) superviscosity coefficient. As a start of the study, decaying turbulence is investigated without any vorticity forcings. Computations are done in a nondimensional form with a unit radius of the sphere (a = 1).

As for the initial condition, amplitude and phase of the spectral components are given randomly to have the same energy spectrum in the form of \( E(n, t=0) \propto n^5 e^{-n^2/2} \) and \( \sum_{n=2}^{N} E(n, t=0) = 1 \), where \( n \) is the total wavenumber of the spherical harmonics. In this study, we make ensemble

averages of 48 runs of which initial conditions are determined with taking account of the spatial symmetry, because the evolution depends on the initial condition. The superviscosity coefficient is fixed at \( \nu_4 = 10^{-6} \) without dimension. Six values of \( \Omega \) are taken as an experimental parameter: 0, 25, 50, 100, 200 and 400. The sphere has \( \Omega/(2\pi) \) rotations per unit time interval. Rotation of the earth is close to \( \Omega = 50 \) with dimension.

A pseudospectral method is used to compute the Jacobian term with a triangular truncation of \( N = 85 \). The Fourth-order Runge-Kutta method is used for time integrations with \( \Delta t = 5 \times 10^{-3} \). All of the computations are done in double precision.

3. Results

3.1. ENERGY AND ENSTROPHY

Time series of the total energy \( E(t) \) and the total enstrophy \( Q(t) \) are obtained as ensemble averages of 48 runs (Fig.1). The total energy decays slowly; it decreases 15 ~ 23 % over a time interval of 5 units. The energy decay rate is larger for larger \( \Omega \). The total enstrophy decays about 87 % for \( \Omega = 0 \) over the time interval. The enstrophy decay rate is smaller for larger \( \Omega \) (72 % for \( \Omega = 400 \)). After \( t \sim 1 \) decay of the total enstrophy is similar to \( t^{-1} \) for small \( \Omega \). Both the sphericity and the rotation have the suppressing effects on the cascade and decaying processes.

Figure 2(a) shows temporal variations of the energy spectrum for the case with no rotation. By \( t = 1 \) there appears a slope close to \( n^{-4} \) in the range of \( 10 < n < 30 \). The slope steepens with time; it is close to \( n^{-6} \) at \( t = 5 \). At this stage there appears another slope close to \( n^{-3} \) for \( n \leq 10 \). The dependency of the energy spectrum at \( t = 5 \) on the rotation rate is shown in Fig.2(b). The total wavenumber with the maximum energy density is larger for larger \( \Omega \) indicating that the upward energy cascade is suppressed by the rotation. As a result the slope of the spectrum is steeper for larger \( \Omega \); it is close to \( n^{-6} \) for \( \Omega = 400 \).
3.2. FLOW FIELD

Figure 2: Evolution of energy spectrum for \( \Omega = 0 \) (a) and \( \Omega \)-dependency of it at \( t = 5 \) (b). Ensemble average of 48 runs.

Figure 3 shows an example of streamfunction fields at \( t = 5 \) for the six values of \( \Omega \), which evolved from the same initial condition. The evolution depends on the rotation rate significantly. A circumpolar vortex of easterly flow emerges for large \( \Omega \) (d~f), while there is no such a polar vortex for \( \Omega = 0 \) (a) and 25 (b). Similar circumpolar vortices are obtained for other initial conditions with large \( \Omega \).

Figure 4 shows an ensemble average of the zonal mean angular momentum; temporal variation of it for \( \Omega = 100 \) (a) and dependency of it on the rotation rate at \( t = 5 \) (b). Results in a hemisphere are shown because they are symmetric with the equator. Although the initial state has no mean zonal flow on average, band structure of alternating mean zonal flow emerges at the initial stage of the evolution (\( t \sim 1 \)) for \( \Omega = 100 \). The easterly flow in high latitudes becomes strong with time, while westerly flow appears in low latitudes as a result of the conservation of total angular momentum. There is a node at \( \phi \sim 45^\circ \) where the mean zonal flow does not change so much for \( 1 \leq t \leq 5 \). The evolution is qualitatively similar for \( \Omega = 50, 100, 200 \) and 400. As shown in Fig.4(b), the intensity of the easterly jet at \( t = 5 \) is not very different for \( \Omega = 100, 200 \) and 400, and the position of the jet shifts into high latitudes as \( \Omega \) increases. The zonal mean angular momentum is roughly constant in middle- and low-latitudes for \( \Omega = 400 \). From the definition this indicates vorticity of the mean zonal flow is nearly zero in these latitudes.

Evolution of the streamfunction field after the emergence of the polar vortex is reminiscent of the nature of Rossby waves. Figure 5 shows the longitude-time sections of the streamfunction at six latitudes \( \phi = \pm 73.9^\circ, \pm 46.0^\circ \) and \( \pm 15.3^\circ \). Westward phase propagation is evident in all the latitudes. The angular velocity is roughly the same (\( \sim 360^\circ \) per unit time interval) in all the latitudes, although the zonal wavenumber of the dominant component is different between the latitudes. Propagations of Rossby wave packets are observed sporadically in time and space.

4. Conclusions

Morphology of two-dimensional decaying turbulence on a rotating sphere was studied with a high-resolution numerical model. Effects of the sphericity and the rotation on the nature of two-
Figure 3: Streamfunction fields at $t = 5$ evolved from the same initial condition for six values of $\Omega$. Contour interval is 0.1 and negative areas are denoted by broken lines. Orthographic projection from $\phi = 90^\circ$ is used.

dimensional turbulence were investigated in detail.

Total enstrophy integrated over the spherical domain decays similarly to $t^{-1}$ for the case with no rotation ($\Omega = 0$). The enstrophy dissipation rate $dQ/dt$ is smaller for larger $\Omega$, and the energy dissipation rate is larger for larger $\Omega$. Energy spectrum changes with time. For the case of $\Omega = 0$, spectral form of $E(n, t)$ is close to $n^{-4}$ in the inertial subrange at the initial stage of the evolution. Later on, there appear two spectral regions for small and large wavenumbers: the spectral form is near to $n^{-3}$ for $n < 10$ and $n^{-5-6}$ for $n > 10$. Temporal variation of the energy spectrum depends on the rotation rate of the sphere. Backward transfer of energy to small wavenumbers is suppressed for large $\Omega$, resulting in the steeper spectrum in the inertial subrange for the larger rotation rate.

Evolution of the flow field depends on the rotation. For $\Omega = 0$ isolated coherent vortices emerge as in the case in Cartesian geometry. As $\Omega$ increases, there appear easterly circumpolar vortices, vorticity of which has the opposite sign of the planetary vorticity. In middle- and low-latitudes, there appears westerly flow as a result of the conservation of total angular momentum. For large $\Omega$, the vortex contracts into polar region with small deviation from the pole, and the flow field is anisotropic in all the latitudes. The westward propagation pattern is observed in longitude-time sections, indicating the excitation of Rossby waves.

Reference

Figure 4: Temporal variation of zonal mean angular momentum for $\Omega = 100$ (a) and $\Omega$-dependency of it at $t = 5$ (b). Ensemble average of 48 runs.

Figure 5: Longitude-time sections of streamfunction (deviation from the zonal mean) for $\Omega = 100$ at $\phi = 73.9^\circ$ (a), $46.0^\circ$ (b), $15.3^\circ$ (c), $-73.9^\circ$ (d), $-46.0^\circ$ (e), and $-15.3^\circ$ (f). Contour interval is 0.15 and negative areas are denoted by broken lines.