Nonlinear Interactions in a Two-layer, Quasi-geostrophic, Low-order Model with Topography

Part II: Interactions between Zonal Flow, Forced Waves and Free Waves

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Abstract

Nonlinear interactions between the zonal flow, topographically forced waves and free baroclinic waves are investigated by using the two-layer, quasi-geostrophic, low-order model constructed in Part I (Yoden, 1983). An idealized topography is given by a Fourier component with the largest scale permitted in the present model.

When the zonal flow is more unstable with respect to a free wave than to the forced wave, there appears a final steady state in which the finite amplitude free wave with a constant phase velocity balances with the marginally stable zonal flow and the forced wave decays out. On the other hand, when the flow is more unstable with respect to the wave component directly coupled with the topography, the flow system has both of the forced and free wave components.

All the wave components are coupled with the topography in least severely truncated case in this paper (two meridional modes and three zonal wavenumbers of \( h \), \( 2h \) and \( 3h \) are permitted). Then the flow system has several types of time-dependent behavior depending on the external parameters such as the external thermal forcing, the frictional dissipation and the static stability: Steady flow with constant forced wave and propagating free wave, periodic or quasi-periodic oscillation and irregular fluctuation.

For the external parameters corresponding to the real atmosphere, there appears an irregular fluctuation with large-amplitude waves. Statistical relation between the zonal flow and waves in the irregular fluctuation is investigated over a long time-span. The flow pattern at each time step is classified into one of three categories in terms of the magnitude of the mean zonal flow. Composite fields in three categories are characterized by the zonality in the high-index state and the moderate state and by the meander of the flow in the low-index state. When the flow is in the low-index state, both of the mean value and vertical shear of the zonal flow are small, the stationary waves have larger amplitudes, and the transient waves have smaller amplitudes compared with the high-index and the moderate states. The structure of the stationary waves in the irregular fluctuation is different from that of the forced wave in the equilibrium solutions in Part I.

1. Introduction

This paper is the second part of a two-part report on the nonlinear interactions between the zonal flow and long waves in a two-layer quasi-geostrophic model with topography. In Part I (Yoden, 1983) a low-order model (i.e., a highly-truncated spectral model) was constructed and the zonal flow-forced wave interaction was investigated by neglecting the components of free baroclinic waves. It was revealed that the multiple stable equilibria as in the barotropic model (Charney and DeVore, 1979) do not exist in the parameter range of the earth's atmosphere. However, the multiplicity of time-dependent solutions were found in the topographically forced planetary wave system. In this Part II free baroclinic waves are included to study the interactions between the zonal flow, topographically forced waves and free baroclinic waves.
From a standpoint of the interactions between the zonal flow and long waves some different theories have been proposed to explain the blocking phenomena (see e.g., Austin, 1980; Treidl et al., 1981 as to recent observational studies). Charney and DeVore (1979) and Charney and Straus (1980) obtained multiple equilibrium states in low-order models with topography and insisted that two stable states correspond to high and low indices in the atmosphere. Solitary Rossby wave theory was applied to the blocking by McWilliams (1980) and Patoine and Warn (1982). Implications of the linear and nonlinear resonance of stationary long waves were discussed by Tung and Lindzen (1979) and Trevisan and Buzzi (1980). For the onset of blocking Frederiksen (1982) considered the instability characteristics of three dimensional flow with stationary long waves.

Numerical experiments have also been performed to study the dynamics of blocking or quasi-stationary waves. Egger (1978) proposed that nonlinear interactions between forced waves and slowly moving free waves lead to a development of the blocking and examined the idea by using barotropic and baroclinic (two-layer) models with a constant topographic forcing. Further development of Egger's work was performed by Schilling (1982). He obtained model-generated blockings in a series of numerical integrations and discussed the flow configuration and energetics. Yao (1980) also studied the energetics for the maintenance of the quasi-stationary waves. Two types of energy cycle were obtained depending on the magnitude of differential heating with latitude. For a small gradient of the heating the flow is less irregular and kinetic energy of the stationary wave is mainly converted from that of the zonal component through the topographic effect in the form of a vertical geopotential flux at the surface. On the other hand, the flow becomes highly irregular for a larger gradient of the heating and the quasi-stationary waves are generated mainly by the baroclinic instability of the forced waves. There are some differences between their numerical models and the present model on the parameterizations of friction and heating, the formulation of the topographic effect and the truncation of spectral components.

The goal of our present study is to investigate the interactions between the zonal flow, forced waves and free baroclinic waves for a better understanding of the blocking phenomena. In Part I it was pointed out that the multiple flow equilibria in the low-order models are not directly related to the atmospheric blocking. However, there is a possibility that time-dependent solutions in the low-order models may give an insight to the nature of the blocking.

The influence of the free baroclinic waves on the equilibrium and time-dependent solutions in the zonal flow-forced wave system will be investigated by a stepwise relaxation of the truncation level. Fig. 1 shows the schematic representation of the interactions permitted in the systems of several truncation level. Three cases are considered in the present study. The surface topography is given by a single component of the lowest zonal wavenumber ($\bar{n}$) with the gravest meridional mode ($m=1$). (In Part II the same notations are adopted as in Part I.) The most simplified system (case 1) with both of the forced and free waves is limited to one meridional mode ($M=1$) and two waves ($N=2$). In this case each wave component interacts with the zonal component independently. If we permit the second meridional mode and harmonics of the lowest wavenumber, it becomes possible to describe the wave-wave interactions. In the case 2 of $M=2$ and $N=2$, there is only one type of wave-wave interaction, in which (1, $\bar{n}$), (2, $\bar{n}$) and (1, 2$\bar{n}$) components are combined with one another by the Jacobian term and the topographic term. Hereafter the first integer in each parenthesis refers to the meridional mode and the second one the zonal wavenumber. In the case 3 of $M=2$ and $N=3$, another type of interaction is possible, i.e., triad interactions between the waves of $\bar{n}$, 2$\bar{n}$ and 3$\bar{n}$. In this truncation level all the wave components interact with the topography of (1, $\bar{n}$) component.

Effects of the truncation are examined in section 2 for three cases shown in Fig. 1. In section 3 dependency on the external parameters such as the external differential heating, the static stability and the frictional time constants is examined by changing the parameter values. We can obtain an irregular fluctuation with a similar energy spectrum to the atmosphere in least severely truncated case and in some ranges of the external parameters. In section 4 some numerical integrations are performed for 2,800 model days to investigate the relation between the zonal flow and long waves statistically. Discussion and conclusion are in sections 5 and 6.
2. Effects of truncation

In this section we will show how the gross features of the interactions between the zonal flow, forced waves and free waves are affected by the truncation (Fig. 1). Ordinary differential equations of the present low-order model are Eqs. (2-8) and (2-9) in Part I. Then the degrees of freedom of the systems are 10 for case 1, 20 for case 2 and 28 for case 3.

a. case 1 ($M=1, N=2$)

In the case of $M=1$, each wave component interacts with the zonal component independently (Fig. 1). Therefore the equilibrium solutions obtained in most severely truncated case (Fig. 3 in Part I) are one part of the equilibrium solutions in the present system when we take all the free wave components equal to zero.

First we examine the linear stability of the equilibrium solutions in the present system. The growth rate $\sigma_r$ is obtained as a real part of complex eigenvalues of the $10 \times 10$ coefficient matrix. The characteristic equation can be rewritten in the product of the sixth-order equation of $a$ for the zonal flow and forced wave part and the forth-order equation for the free wave part. The stability properties for the zonal flow and forced wave part are identical to the result obtained in the case of $M=1$ and $N=1$ in Part I. Because the forced wave and the topography do not directly interact with the free wave, the characteristic equation for the free wave contains only the zonal flow components of the equilibrium solutions. Therefore the stability of the equilibrium solutions with respect to the perturbations of the free wave is determined by the conventional baroclinic stability analysis of the zonal flow.

The growth rate of the most unstable perturbation embedded in the equilibrium solution of $\zeta=3$ is shown in Fig. 2 for the zonal wavenumber $\tilde{n}=1-12$. Note that the ordinate is the external forcing parameter $\theta_A^*$ (not the vertical shear of the zonal wind), although the vertical shear depends on $\theta_A^*$ linearly only in the Hadley solution (Eq. (3-15) in Part I). The growth rate for $\zeta=3$ must be excluded (broken lines in the figure), because the perturbation of $\zeta=3$ feels the surface topography and then it is not free. The Hadley solution is most unstable with respect to the perturbation of $\zeta=9$ in the range of $0.016 \leq \theta_A^* \leq 0.018$, $\zeta=8$ in $0.02 \leq \theta_A^* \leq 0.048$ and $\zeta=7$ in $0.05 \leq \theta_A^*$. Both of the equilibrium solutions with wave components (W1 and W2 solutions shown in Fig. 3 in Part I) are unstable with
Fig. 2  Linear stability of the equilibrium solutions ($\tilde{\eta}=3$) with respect to free waves with zonal wavenumber $\tilde{\eta}=1-12$ (abscissa). Growth rate of the most unstable perturbation is contoured. In dimensional, $\sigma_r=0.1$ corresponds to e-folding time of 27 hours.

Fig. 3  Time-evolution of the zonal and wave components in case 1. Initial condition is the W1 solution with small perturbation ($\theta_{\text{a}*}=0.2$). 1,000 nondimensional time corresponds to 112.5 days.
respect to the free waves of $4 \leq \tilde{n} \leq 11$ and most unstable for $\tilde{n} = 7$. Because the zonal flow of the W1 solution does not depend on $\theta_A^*$ so much, the growth rate is almost independent of $\theta_A^*$ for the W1 solutions.

Almost all the equilibrium solutions obtained in the zonal flow-forced wave system are baroclinically unstable with respect to a perturbation of free wave. If a free wave perturbation is added to the unstable equilibrium state, it will grow up and interact with the zonal flow. Then the zonal flow will change and the forced wave will also change in time.

Some numerical integrations are performed to elucidate the time-dependent behavior of each component. Time-variations of the zonal flow and the waves are presented in Fig. 3. The zonal wavenumber of the topography is $\tilde{n} = 3$ and that of the free wave is $\tilde{n}' = 7$. The external forcing parameter $\theta_A^*$ is fixed to 0.2 and the initial state is the W1 solution with a small perturbation of the free wave. Initially the free wave grows up with eastward phase-propagation of 16 m/s in dimensional value and the zonal-mean meridional temperature difference $\theta(1, 0)$ decreases. These features are in agreement with those predicted by the linear baroclinic instability theory. Simultaneously vertically averaged zonal flow $\phi(1, 0)$ increases and the amplitudes of the forced wave $\Phi(1, 3)$ and $\theta(1, 3)$ decrease due to the changes in the topographic and Jacobian terms.

After some fluctuations of each component (about 500 non-dimensional time steps), the zonal flow converges to a small value and the forced wave decays out. The free wave converges to a constant amplitude with the westward phase velocity of 0.92 m/s (see also Table I). Because the zonal flow in the lower layer is equal to zero (i.e., $\phi(1, 0) - \theta(1, 0) = 0$), there is no effect of the topography. The same final steady state of the weak zonal flow and free wave with constant amplitude and constant phase velocity is also obtained for other initial conditions (e.g., the Hadley solution and the W2 solution).

Similar final steady states are obtained for other forcing parameter $\theta_A^*$ (Table I). It is found that the zonal flow components ($\phi(1, 0)$ and $\theta(1, 0)$), the phase difference between the upper and lower layers ($\Delta \phi$) and phase velocity ($c$) of the free wave are independent of the external forcing. As the forcing parameter $\theta_A^*$ increases, only the amplitude of the free wave ($\phi(1, 7)$ and $\phi(1, 7)$) becomes large to compensate the increment of differential heating.

Note that the magnitudes of the zonal flow are equal to those at the critical point where the Hadley solution becomes baroclinically unstable with respect to the free wave. The value of $\theta_A^*$ at the critical point is 0.01813 for $\tilde{n}' = 7$, which is substituted into the equation (3-15) in Part I to obtain the critical value of $\phi(1, 0) = \theta(1, 0) = 0.01716$. At the critical point the phase velocity of the perturbation is equal to zero in the case without friction (see the baroclinic stability analysis in the two-layer model by e.g., Pedlosky,

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<th>$\theta(1, 0)$</th>
<th>$\phi(1, \tilde{n}')$</th>
<th>$\theta(1, \tilde{n}')$</th>
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1979). But in the present model the phase velocity has a small negative value due to the friction and diabatic heating of the eddy component.

If we vary the zonal wavenumber of the free wave in the range of $4 \leq \hat{n}' \leq 11$, we obtain a final steady state in which the zonal flow with the critical value balances with the finite-amplitude free baroclinic wave. Dependency of the phase velocity (c) on the zonal wavenumber reminds us of the dispersion relation of free Rossby waves. On the other hand, when we put $\hat{n}' \leq 2$ or $\hat{n}' \geq 12$, the dependent variables asymptotically approach the W1 solution as in the zonal flow-forced wave system (Part I).

In this truncation level of $M=1$ and $N=2$, there finally appears either the W1 solution with forced wave or the final steady state with free wave. There does not appear such a state that both of the forced and free waves exist together and interact with the zonal flow.

b. case 2 ($M=2$, $N=2$)

If we permit the second meridional mode and the harmonic of the lowest zonal wavenumber, we obtain a dynamical system with 20 degrees of freedom, which contains the wave-wave and wave-topography interactions between $(1, \hat{n})$, $(2, \hat{n})$ and $(1, 2\hat{n})$ components (broken line in Fig. 1).

Time-variation of vertically averaged zonal flow and wave components is shown in Fig. 4. The zonal wavenumber of the topography is $\hat{n}=3$ and the harmonic wavenumber $2\hat{n}=6$. The initial condition is the W1 solution with small perturbations of every components. The external forcing $\theta_A^*$ is again 0.2.

Variations of the zonal flow and forced wave of the first meridional mode resemble those obtained in case 1 (Fig. 3). Initially $\phi(1,0)$ increases and the forced wave becomes weak. After some fluctuations the zonal flow converges to a weak state and the forced wave decays out. All the free wave components grow up first due to the baroclinic instability of the zonal flow. However, as the zonal component of $\phi(1,0)$ converges to a small value, all other components

![Fig. 4](image)

As in Fig. 3 for the truncation level of case 2. Only $\phi$ components are presented.
except for $\phi(2, 6)$ decay out. The final steady state of a weak zonal flow and free wave of constant amplitude and constant phase velocity is similar to that obtained in case 1. The meridional mode of the free wave, however, is the secondary mode which has no interaction with the topography. Similar final steady states are obtained for other external forcing parameter $\theta_A^*$ and only the amplitude of the free wave increases with increasing $\theta_A^*$.

For the topographic wave of $\bar{n}=4$ there appears a similar final steady state but for $\bar{n}=5$ corresponding steady state does not appear. All the variables go on fluctuating irregularly in the $\bar{n}=5$ case. Meanwhile, if the static stability parameter $\sigma_0$ is changed to a larger value (1.25 times or 1.5 times of the value given in Part I), the final steady state is obtained as well. However, when the static stability exceeds a critical value, e.g., doubled $\sigma_0$ ($2\delta/\Delta P = 60$ K/500 mb), there appears an irregular fluctuation. In these situations $(1, 2\bar{n})$ component is more unstable than $(2, 2\bar{n})$ component and interacts with the topography and other waves as well as the zonal flow. The linear baroclinic instability theory (Eady, 1949) shows that stability criterion is proportional to the product of the static stability and the square of wavenumber ($= \sigma_0 \times (m^2 + n^2)$). Therefore the most unstable wavenumber shifts toward a small value with increasing the static stability $\sigma_0$.

In the present truncation level, there appears either a final steady state or an irregular fluctuation. Only $(1, 0)$ and $(2, 2\bar{n})$ components have non-zero values in the final steady state and then there do not exist the wave-wave and wave-topography interactions. On the other hand, in the irregular fluctuation all the components fluctuate with interactions of wave-zonal flow, wave-wave, zonal flow-topography and wave-topography. The alternative of two flow regimes depends on the horizontal scale of the topography (and therefore that of the forced and free waves) and the static stability but is independent of the external forcing parameter.

c. case 3 ($M=2, N=3$)
Because $(2, 2\bar{n})$ component is not coupled with

![Fig. 5 Time-variation of the zonal and wave components in case 3 (only $\phi$ components are presented). $\theta_A^*=0.2$. 2,500 time steps from $t=5,001$ to $t=7,500$ are presented (in dimensional 281.3 days).](image-url)
the topography in case 2, the final steady state is possible. In case 3 of \( M = 2 \) and \( N = 3 \), however, all the wave components are coupled with the topography by the \( \bar{n} = 2 \bar{n} \) interaction and the triad interactions between \( \bar{n}, 2\bar{n} \) and \( 3\bar{n} \). Therefore, if one of the wave components is not zero, another wave component will be induced by the wave-topography interaction. Final steady states like those in cases 1 and 2, in which there is no topographic effect, are not possible in the present case.

Time-variations of the vertically averaged zonal flow and wave components are presented in Fig. 5 for the same external parameters as those in Figs. 3 and 4. There appears the same periodic solution for the initial conditions of the Hadley, W1 and W2 equilibrium solutions with small perturbations. As in cases 1 and 2, the zonal flow of the first meridional mode \( \phi(1,0) \) has a small magnitude and the wave of the smallest scale \( \phi(2,9) \) has the largest wave amplitude. However, other wave components also have non-zero values by the triad interactions. The amplitude of \( \phi(2,6) \) is about a half of that of \( \phi(2,9) \) and other four components have rather small amplitudes (see also Table 3).

The zonal flow of the first meridional mode \( \phi(1,0) \) fluctuates with the periods of 7.6 days and 85 days. The amplitude of the wave components have the same periodicity. Wave components of \( \phi(2,6) \) and \( \phi(1,9) \) propagate eastward with the period of 85 days. The characteristics of the time-variations are different depending on the external parameters such as the external forcing \( \theta_A^* \), the static stability \( \sigma_0 \) and the frictional coefficients \( k \) and \( k' \). This dependency will be discussed in the next section.

Effects of the truncation were investigated in three cases with different truncation level. It is not possible for the system of case 1 \( (M = 1, N = 2) \) to depict such a state that both of the forced and free waves exist and interact with the zonal flow. In case 2 \( (M = 2, N = 2) \), such a state is obtained for a small-scale topography \( (\bar{n} \geq 5) \) or a large static stability \( (\sigma_0 \times 2) \). For the parameter values given in Part I (which are typical in the atmosphere), however, there appears the final steady state in which the zonal flow and a free wave have non-zero values and the topography has no contribution. On the other hand, both of the forced and free waves exist and interact with each other in case 3, because all the waves are coupled with the topography (Fig. 1). At least 28 degrees of freedom (case 3) are necessary for the system to depict the coexistence of both the forced and free waves and the interactions with each other. In the next two sections we will investigate the interactions between the zonal flow and these waves at the truncation level of case 3.

3. Dependency on external parameters

In Part I we tentatively fixed the external parameters such as the frictional and heating coefficients, the static stability and the amplitude of the topography. As a control parameter, only the external forcing parameter \( \theta_A^* \) was varied from 0 to 0.2 (The corresponding temperature difference across the channel is from 0 K to 150 K at the radiative equilibrium state.). In this section we will show the effects of \( \theta_A^*, \sigma_0, k \) and \( k' \) on the behavior of the zonal flow and waves. The zonal wavenumber of the topography is fixed to \( \bar{n} = 3 \); wavenumbers permitted in the present model are 3, 6 and 9.

The characteristics of the time-variations are different depending on the external parameters. In Table 2 the time-dependency is classified into four categories: Steady flow with wave components \( (\bar{s}) \), periodic oscillation \( (P) \), quasi-periodic oscillation \( (Q) \) and irregular fluctuation \( (I) \). Some typical variations of the zonal component \( \phi(1,0) \) for each category are presented in Fig. 6. Here quasi-periodic oscillation is subjectively discriminated from irregular fluctuation by looking over the time series of the amplitudes as in Fig. 6.

For a wide range of \( \theta_A^* \) there appears a periodic oscillation in the cases with external parameters given in Part I. As the frictional parameters decrease, there appears a quasi-periodic oscillation or an irregular fluctuation instead of the periodic oscillation. The time-dependency is influenced by the external forcing parameter \( \theta_A^* \) and frictional parameters \( k \) and \( k' \). However, as shown in Table 3, the dominant components such as \( (1, 0) \), \( (2, 6) \) and \( (2, 9) \) are insensitive to the changes in these external parameters. Here the mean zonal flow \( \bar{\phi}(m, 0) \) and the mean amplitude of the wave \( \bar{\phi}(m, n) \) are given by

\[
\bar{\phi}(m, 0) = \frac{1}{N} \sum_{i=1}^{N} \phi_{A_m}(t_0 + i\Delta t)
\]

\[
\bar{\phi}(m, n) = \frac{1}{N} \sum_{i=1}^{N} [\psi_{K_m}(t_0 + i\Delta t)^2]^{1/2}
\]

The dependency of time-dependent behavior of
Table 2 Classification of time-dependent behavior for some combinations of external parameters. Four categories are denoted by $S$: steady flow with propagating wave, $P$: periodic oscillation, $Q$: quasi-periodic oscillation, and $I$: irregular fluctuation. Subscripts 5 and a-j denote corresponding figure in Fig. 5 and Fig. 6(a)-(j).

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### (b)

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Fig. 6 Four types of time variation of the zonal component $\phi(1, 0)$. External parameters in (a)–(i) are listed in Table 2. Steady flow (5): (g), periodic oscillation (P): (c), (j), quasi-periodic oscillation (Q): (a), (d), and irregular fluctuation (I): (b), (e), (f), (h), (i).
Table 3  Mean values of the zonal flow and amplitudes of wave components. Averaged period is from 3,750 to 7,500 in non-dimensional time.

<table>
<thead>
<tr>
<th>$\theta_0$</th>
<th>$k\cdot k'$</th>
<th>$\sigma_0\times$</th>
<th>$\bar{\phi}(1,0)$</th>
<th>$\bar{\phi}(2,0)$</th>
<th>$\bar{\phi}(1,3)$</th>
<th>$\bar{\phi}(2,3)$</th>
<th>$\bar{\phi}(1,6)$</th>
<th>$\bar{\phi}(2,6)$</th>
<th>$\bar{\phi}(1,9)$</th>
<th>$\bar{\phi}(2,9)$</th>
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</thead>
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<tr>
<td>0.02</td>
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<td>1.0</td>
<td>177</td>
<td>0</td>
<td>3</td>
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<td>43</td>
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<td>&quot;</td>
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<td>6</td>
<td>7</td>
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<td>&quot;</td>
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<td>14</td>
<td>191</td>
<td>9</td>
<td>584</td>
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<tr>
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<td>&quot;</td>
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<td>1.0</td>
<td>241</td>
<td>0</td>
<td>13</td>
<td>7</td>
<td>8</td>
<td>106</td>
<td>5</td>
<td>484</td>
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<tr>
<td>&quot;</td>
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<td>&quot;</td>
<td>284</td>
<td>0</td>
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<td>233</td>
<td>13</td>
<td>511</td>
</tr>
<tr>
<td>&quot;</td>
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<td>&quot;</td>
<td>315</td>
<td>0</td>
<td>72</td>
<td>55</td>
<td>93</td>
<td>265</td>
<td>36</td>
<td>592</td>
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<td>0.1</td>
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<td>6</td>
<td>9</td>
<td>8</td>
<td>0</td>
<td>1</td>
<td>452</td>
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<tr>
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<td>&quot;</td>
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<td>0</td>
<td>13</td>
<td>7</td>
<td>8</td>
<td>106</td>
<td>5</td>
<td>484</td>
</tr>
<tr>
<td>&quot;</td>
<td>1.25</td>
<td>&quot;</td>
<td>270</td>
<td>3</td>
<td>189</td>
<td>185</td>
<td>188</td>
<td>150</td>
<td>189</td>
<td>311</td>
</tr>
<tr>
<td>&quot;</td>
<td>1.5</td>
<td>&quot;</td>
<td>357</td>
<td>18</td>
<td>219</td>
<td>213</td>
<td>222</td>
<td>207</td>
<td>200</td>
<td>194</td>
</tr>
<tr>
<td>&quot;</td>
<td>2.0</td>
<td>&quot;</td>
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<tr>
<td>&quot;</td>
<td>4.0</td>
<td>&quot;</td>
<td>795</td>
<td>−0</td>
<td>56</td>
<td>111</td>
<td>41</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

Solutions on the static stability $\sigma_0$ is shown in Table 2-(b). For a small $\sigma_0$ there appears a steady flow (Fig. 6-(g)), in which the westward propagating wave $\phi(2,9)$ is dominant and the forced wave $\phi(1,3)$ is stationary. As the static stability increases, there take place transitions from the steady flow to the periodic oscillation and from the periodic oscillation to the irregular fluctuation (Figs. 6-(c), (h) and (i)). For a large $\sigma_0$, there appears another type of the periodic oscillation (Fig. 6-(j)).

With increasing $\sigma_0$, the magnitude of the zonal component with the first meridional mode increases and wave components of small zonal wavenumbers ($\bar{n}=3$ and 6) become dominant (Table 3). For the case of $\sigma_0\times 4$, $\phi(1,3)$, $\phi(2,3)$ and $\phi(1,6)$ have large amplitudes. Difference of the periodic solutions between the cases of $\sigma_0\times 1$ and $\sigma_0\times 4$ is clearly seen in the mean values of the wave amplitudes. The dependency of the dominant wavenumber on the static stability is reminiscent of the linear baroclinic instability theory (Eady, 1949) mentioned in the previous section: The stability criterion and the most unstable mode are dependent on the product of the static stability and the square of the wavenumber.

For the external parameters given in Part I, the amplitude of the wave (1, 3) with the same scale as the topography is rather small compared with (2, 6) and (2, 9) components. This is not the case in the real atmospheric circulation, where ultra-long waves in general have larger amplitude than long waves in the mid-latitude troposphere. External parameters must be changed in the present system of $N=3$ in order to obtain a similar energy spectrum like that of the atmosphere.

As shown in Table 3, mean amplitudes of wave components of small zonal wavenumbers become large with increasing the static stability $\sigma_0$. Although $\sigma_0=5.64\times 10^{-2}$ (in dimensional $2\bar{\theta}/D\bar{\theta}=30$ K/500 mb) adopted in Part I is a typical value in mid-latitudes, a larger value is usual in winter (see e.g., Tomatsu, 1979). Charney and Strauss (1980) adopted the value $2\bar{\theta}/D\bar{\theta}=43.6$ K/500 mb and Yao (1980) did 31.8 K/500 mb. Increase of the static stability parameter to 1.25 times (37.5 K/500 mb) or 1.5 times (45 K/500 mb) is not far from the atmospheric conditions.

Fig. 7 shows the time-variations of vertically averaged zonal flow and wave components for the case of the increased static stability ($\sigma_0\times 1.25$). All the components have large amplitudes and fluctuate in a highly irregular manner. The behavior of the zonal flow and the forced wave does not resemble to that obtained in the zonal flow-forced wave system.

The zonal component $\phi(1, 0)$ fluctuates around the mean value (denoted by broken line) and sometimes has extremely large or small values. Such extreme states continue during the period ranging from several days to several ten days. The amplitude and the phase of each wave also vary in connection with the fluctuations of the zonal flow and other waves. After $t=7,000$, for example, $\phi(1, 0)$ takes extremely small values for
140 nondimensional time (about 15 days). In this period \( \phi(1, 3) \) has a small amplitude but the phase is rather quasi-stationary. While, \( \phi(1, 9) \) has a large amplitude and is quasi-stationary.

In the next section we will clarify the relation between the zonal flow and waves statistically over a long time-span. We have so far used the term 'forced wave' to the wave (1, 3) because it has the same sizes in both \( x \) and \( y \) directions as those of the topography. But this usage is not the conventional one except when the wave happens to be steady and stationary. In the present truncation level of case 3, all the wave components are affected by the topography and the topographic effects vary in time due to the variation of flow itself. Therefore the 'forced wave' is not appropriate in the present problem and is not used in the next section. We will define 'stationary wave' as a mean wave over a time and direct our attentions to the relation between the zonal flow and the stationary waves.

4. Zonal flow and stationary waves in irregular fluctuations

Numerical integrations for three combinations of external parameters (Table 4) are performed for 25,000 time steps (in dimensional 2,813 days). Three initial conditions are taken in each case, i.e., W1, W2 and Hadley solutions with small perturbations. Mean value (MV) and standard deviation (SD) of \( \phi(1, 0) \) over 23,750 time steps except for initial stage are listed in Table 4. As the forcing parameter \( \theta_A^* \) increases or the static stability \( \sigma_0 \) increases, MV and SD also increase with a few exceptions. SD is smaller than MV in the cases of \( \sigma_0 \times 1.25 \) but is comparable with MV in the case of \( \sigma_0 \times 1.5 \) and \( \theta_A^* = 0.2 \). In each case MV and SD have variations of 1–15% depending on the initial conditions.

We classify the flow at each time step into three categories in terms of \( \phi(1, 0) \) component. The threshold values are MV±SD as denoted by
Table 4 Mean value (MV) and standard deviation (SD) of \( \phi(1, 0) \) for 12 numerical integrations. Period of the statistics is from 1,251 to 25,000 (in dimensional 2,672 days). Figure in parenthesis is the result for a control experiment without surface topography. All the values are multiplied by 10^4.

<table>
<thead>
<tr>
<th>( \theta_{A1} )</th>
<th>( \sigma_0 \times 1.25 )</th>
<th>( \sigma_0 \times 1.5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>W1</td>
<td>W2</td>
</tr>
<tr>
<td>0.15 MV</td>
<td>290</td>
<td>288</td>
</tr>
<tr>
<td>SD</td>
<td>182</td>
<td>173</td>
</tr>
<tr>
<td>0.2 MV</td>
<td>308 (288)</td>
<td>263 (291)</td>
</tr>
<tr>
<td>SD</td>
<td>219 (21)</td>
<td>234 (21)</td>
</tr>
</tbody>
</table>

Table 5 Zonal components in three categories. All the values are multiplied by 10^4. Total number in each category \( N' \) is also listed.

<table>
<thead>
<tr>
<th>Category</th>
<th>( N' )</th>
<th>( \phi(1, 0) )</th>
<th>( \theta(1, 0) )</th>
<th>( \phi(2, 0) )</th>
<th>( \theta(2, 0) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>3717</td>
<td>3471</td>
<td>3375</td>
<td>629</td>
<td>604</td>
</tr>
<tr>
<td>M</td>
<td>16486</td>
<td>16579</td>
<td>16799</td>
<td>313</td>
<td>276</td>
</tr>
<tr>
<td>L</td>
<td>3547</td>
<td>3700</td>
<td>3576</td>
<td>-52</td>
<td>-119</td>
</tr>
</tbody>
</table>

Fig. 8 Composite fields of stream function in three categories. \( \sigma_0 \times 1.25, \theta_{A1} = 0.2 \) and the W2 initial condition.

dotted lines in Fig. 7. We term the state with \( \phi(1, 0) > MV + SD \) high-index state (H), the state with \( MV + SD \geq \phi(1, 0) \geq MV - SD \) moderate state (M), and the state with \( MV - SD > \phi(1, 0) \) low-index state (L). Mean zonal component \( \phi(1, 0) \) fluctuates irregularly and transits between these categories. The ratio of the time span in each category is almost (H):(M):(L) = 15:70:15 independently of the initial conditions (Table 5).

The composite fields of stream function in three categories are shown in Fig. 8 for the case of \( \sigma_0 \times 1.25 \) and \( \theta_{A1} = 0.2 \) (the W2 initial conditions). In the high-index state (H) the flow pattern is nearly zonal in both layers. In the moderate state (M) the zonal flow component is dominant in the upper layer but there is almost no motion in the lower layer. In the low-index state (L) the flow pattern is wave-like and the ridge of the dominant component is located in the western slope of the mountain. The zonal flow in the lower layer is easterly in this category. The distinctive feature of the composite flow pattern is the zonality in (H) and (M) and the meander in (L).

Magnitude of the zonal components averaged in each category is listed in Table 5. Of course, the vertically averaged zonal flow \( \phi(1, 0) \) has the largest value in the category (H) and the smallest (negative) value in (L). The meridional difference of the zonal potential temperature \( \theta(1, 0) \) (i.e., mean vertical shear of the zonal wind by the thermal wind relation) also decreases with the
Fig. 9. Amplitude and phase of the stationary waves in three categories. Three independent runs with different initial conditions are designated by symbols as follows; W1 (O), W2 (A) and Hadley (+). $\phi(m,n)$ component is denoted by a solid line and $\theta(m,n)$ by a broken line. Amplitude is multiplied by $10^3$. $\sigma_0 \times 1.25$ and $\theta_A^{*} = 0.2$.

decrease of $\phi(1,0)$; $\theta(1,0)$ in (L) is about 80% of that in (H). On the other hand, zonal components of the second meridional mode ($\phi(2,0)$ and $\theta(2,0)$) are very small in all the categories.

The stationary waves are defined in three categories; cosine and sine components are given by

$$\bar{\phi}(m,n) = \frac{1}{N'} \sum_{k=1}^{N'} \phi k_m^n$$

$$\bar{\psi}(m,n) = \frac{1}{N'} \sum_{k=1}^{N'} \psi k_m^n$$

where $N'$ is the total number in the category. Fig. 9 shows the amplitude and phase of the stationary waves in three categories. Only the lowest mode (1, 3) with the same scale as the topography has large amplitude. Other components have small amplitudes with a few exceptions and have phase variations of $40^\circ$–$70^\circ$ depending on the initial conditions. The stationary wave of $\phi(1, 3)$ has a large amplitude in (H) and (L) and small value in (M). Note that the phase in (L) is very close to each other ($\approx 8^\circ$) in the three numerical integrations. While, $\theta(1, 3)$ has a large amplitude and nearly the same phase in (L) and (M). The phase difference between $\phi(1,3)$ and $\theta(1,3)$ is $115^\circ$ in (L) and very small in (H). In the category (L) the heat flux by this stationary wave is southward (counter-gradient for the zonal mean temperature).

We obtain similar result in other cases in Table 4. The composite fields of stream function in the upper layer are shown in Figs. 10. For a smaller forcing parameter ($\theta_A^{*} = 0.15$, Fig. 10-(1)), the zonal flow becomes weak a little in (H). In (L) similar flow patterns are obtained like those in Fig. 8. The amplitude and the phase of (1,3) components are not influenced by the external forcing parameter; although there are some differences in small scales. In the case of the increased static stability ($\sigma_0 \times 1.5$, Fig. 10-(2)), stationary waves of small scales have small amplitude and the flow pattern is smoothed out. The amplitude of the (1, 3) components in (L) increases a little with increasing the static stability and the phase of $\phi(1,3)$ is shifted westward about $60^\circ$ without a change in the phase of $\theta(1,3)$.

A control experiment is performed to verify the effect of surface topography. We set the amplitude of topography equal to zero in the case of $\sigma_0 \times 1.25$ and $\theta_A^{*} = 0.2$. Mean value and standard deviation of $\phi(1,0)$ are listed in Table 4. MV has nearly the same value as in the case with topography but SD is small about one order. The surface topography produces a large variability of the vertically averaged zonal flow $\phi(1,0)$ in the present model. Composite fields for the control experiment are shown in Fig. 10-(3). The amplitude of all the stationary waves is small and the phase of them is dependent on the initial conditions. Especially the amplitude of $\phi(1,3)$ is small about one order or more com-
pared with the cases with the topography.

5. Discussion

Almost all the equilibrium solutions obtained in the zonal flow-forced wave system in Part I are baroclinically unstable with respect to a perturbation of free wave with $4 \leq \eta' \leq 11$ (Fig. 2). When a perturbation of the free wave is added to the unstable equilibrium states, the free wave grows up and the forced wave decays out as shown in Fig. 3. Finally there appears a steady state in which finite amplitude free wave with a constant phase velocity balances with the marginally stable zonal flow. This steady state is independent of the forcing parameter $\theta_A^*$, and has no topographic effect because the zonal flow in the lower layer is zero. In case 2 with the second meridional mode and the harmonic of the lowest zonal wavenumber, the wave-wave or wave-topography interaction is possible between $(1, \tilde{n})$, $(2, \tilde{n})$ and $(1, 2\tilde{n})$ components. However, we obtained a similar final steady state as in case 1 with only $(1, 0)$ and $(2, 2\tilde{n})$ components for the external parameters given in Part I and the zonal wavenumber $\tilde{n} = 3$ and 4. There is again no topographic effect because the zonal flow in the lower layer is zero and the amplitudes of $(1, \tilde{n})$, $(2, \tilde{n})$ and $(1, 2\tilde{n})$ are also zero.

These final steady states are reminiscent of the baroclinic adjustment process proposed by Stone (1978). He hypothesized that baroclinic waves adjust the meridional gradient of the zonal mean temperature so as to keep it just above the threshold value for instability in a two-layer quasi-geostrophic model, and he obtained an observational result that the meridional temperature gradient in the northern troposphere is consistent with the hypothesis in all the seasons. The final steady states obtained in section 2 reveals that the baroclinic adjustment process operates completely in these cases of the present low-order model.

In contrast to the final steady states in cases 1 and 2, all the wave components have non-zero values in case 3, because all the wave components are coupled with the topography by the $\tilde{n} - 2\tilde{n}$ interaction and the triad interactions. There appear various flow patterns (i.e., a steady flow, a periodic oscillation and an irregular fluctuation) depending on the external parameters such as the external forcing parameter $\theta_A^*$, the frictional coefficients $k$ and $k'$, and the static stability $\sigma_0$ (Table 2 and Fig. 6). Dependency of the flow patterns on these parameters is consistent with that in the rotating annulus experiments with differential heating and bottom topography (Leach, 1981; Jonas, 1981) and with that in the numerical experiments by Yao (1980): The flow becomes irregular with increasing the Taylor number ($\sim k^{-2}$) or with increasing the thermal
Rossby number (≈ θₜ* or σ₀).

The wave component (1, 3) with the same scale of the topography fluctuates like a standing oscillation (Fig. 5) or in a highly irregular manner (Fig. 7). The behavior of the wave does not resemble that in the zonal flow-forced wave system (Fig. 10 in Part I). Isolated zonal flow-forced wave systems as in Part I and Charney and Straus (1980) are not sufficient to depict the behavior of forced waves when the flow system is more unstable with respect to another disturbance with different scale (in the present case baroclinic waves of 2h and 3h). Charney and DeVore (1979) hypothesized that the baroclinic instability produces additional forcing which ultimately drives the flow system from one metastable equilibrium state to another. However, the present two-layer model which contains the baroclinic waves explicitly does not depict such a transition from one metastable state to another.

When the static stability σ₀ was increased to 1.25 times or 1.5 times, we obtained an irregular fluctuation of which the energy spectrum is similar to that in the real atmosphere. Marcus (1981) examined the effects of truncation in a problem of thermal convection in a sphere. His numerical examples indicate that, as long as the kinetic energy spectrum decreases with wavenumber, a truncation gives a qualitatively correct solution. It is not possible to apply his conclusion directly to our model. However, it is thought that the present model with the truncation level of case 3 and the increase of the static stability can depict a qualitatively correct behavior of the zonal flow and the waves in the atmosphere.

Irregular fluctuations obtained in some numerical integrations in the model with an increased static stability were analyzed in section 4 to elucidate the relation between the zonal flow and the stationary waves. However, the transient waves in each category were not mentioned there. Mean amplitude of the transient wave in each category is given by

\[ \bar{\varphi}_n(m, n) = \frac{1}{N'} \sum_{N'} \left[ (\varphi_{K,m,n} - \bar{\varphi}_n(m, n))^2 + (\varphi_{L,m,n} - \bar{\varphi}_n(m, n))^2 \right]^{1/2} \]  (5-1)

The mean amplitudes of the transient waves are rather large compared with those of the stationary waves. However, transient wave components in the low-index state (L) are smaller than those in the moderate state (M) with some exceptions. \( \bar{\varphi}_n(1, 3) \) component in three categories are as follows; (H): 0.0313, (M): 0.0323 and (L): 0.0293 (σ₀×1.25 and \( \theta_{1,3} = 0.2 \)). In the 9 numerical integrations the decrease of the amplitude in (L) is 5–12% of that in (M). This decrease is consistent with the decrease of mean vertical shear \( \bar{\theta}(1, 0) \) mentioned in section 4.

If the low-index state is related to the blocking phenomena, appearance of such an extreme state is not periodic but unexpectedly (Fig. 7). Irregularity of the present system is more complicated than the 'deterministic nonperiodic flow' (the strange attractor) in the Lorenz's system of three ordinary nonlinear differential equations (Lorenz, 1963). Lorenz found that nonperiodic solutions are ordinarily unstable with respect to small perturbations, so that slightly differing initial states can evolve into considerably different states. His conclusion about the predictability may be applied to the present system. Namely, the prediction of the 'blocking state' (low-index state) for a very long range is impossible unless the initial conditions are known exactly. In Lorenz's study (1965) of the predictability with a 28-variable model, which is essentially identical to the present model except for the topography, the time required for initial errors such as observational errors to grow to intolerable errors is in the range from a few days to a few weeks.

6. Conclusion

Nonlinear interactions between the zonal flow, topographically forced waves and free baroclinic waves were investigated by using the two-layer, quasi-geostrophic, low-order model in a mid-latitude \( \beta \)-plane (Yoden, 1983). An idealized surface topography was included in the model by retaining only one Fourier component of the zonal wavenumber \( \tilde{n} = 3 \) with the greatest meridional mode.

First of all, effects of the truncation were examined by a stepwise relaxation of the truncation level. It was revealed that at least 28 degrees of freedom (case 3 in Fig. 1) are necessary for the system to depict the coexistence of both of the forced and free waves and interactions with each other. For more severely truncated cases, there appears a final steady state with no forced waves, because the baroclinic adjustment process (Stone, 1978) operates completely and the topographic effect is absent.

For the system in case 3, there appear several types of time-dependent flow patterns depending on the external parameters such as the external thermal forcing, the frictional coefficients and
the static stability: Steady flow, periodic or quasi-periodic oscillation and irregular fluctuation. There appears an irregular fluctuation with large amplitude waves for the external parameters corresponding to the real atmosphere.

Irregular fluctuations obtained in the numerical integrations over a long time-span were analyzed to elucidate the relation between the zonal flow and stationary waves. The flow pattern at each time step was classified into one of three categories, i.e., the high-index state (H), the moderate state (M) and the low-index state (L), in terms of the vertically averaged zonal flow $\phi(1,0)$. The ratio of the time span in each category is 15:70:15 for 9 cases in Table 4 when the threshold values are taken as the mean value (MV)± standard deviation (SD).

The composite fields of stream function are characterized by the strong and moderate zonal flows in (H) and (M) and the meander of the flow in (L). In the low-index state (L) the dominant stationary wave of (1, 3) with the same scale as the topography has a larger amplitude than in (H) and (M). The ridge is located in the western slope of the topography. The effects of the surface topography on this stationary wave is obvious from the control experiment without the topography (Fig. 10). The stationary waves in the irregular fluctuation are different from the forced waves in the equilibrium solutions (Fig. 4 in Part I) in their magnitude and structure. The difference between the equilibrium solutions and the time-averaged states was already pointed out by Yao (1980).

In summary, the present model similar to the atmospheric conditions is characterized by the irregularity (or transiency): The flow varies irregularly and transits between the categories. When the flow is in a low-index state, both of the mean value and the vertical shear of the zonal flow decrease, the stationary wave has a larger amplitude, and free waves have smaller amplitudes.

In the present study only the effect of the topography was investigated. However, the land-sea distributions may play an important role in producing the forced planetary waves. Simple representation of the diabatic heating due to the land-sea distributions will be incorporated into the present model by using the Newtonian heating with longitudinal differential equilibrium temperature. It is a future problem to investigate the effect of the land-sea distributions and the coupled effect with the topography by using a simple low-order model.

Acknowledgements

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The computations were performed with the use of the FACOM M-200 computer at the Data Processing Center of Kyoto University.

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地形を含む二層準地衡低次モデルにおける非線型相互作用

II：帯状流－強制波－自由波間の相互作用

余田成男
京都大学理学部地球物理学教室

第Ⅰ部で作成した二層・準地衡風近似の低次モデルを用いて、帯状流と地形による強制波および傾圧不安定波間の非線型相互作用を調べた。理想化された地形として、このモデルで許されるいちばん大きな波成分のみの地形を入れた。

帯状流が、地形と直結しつづいた強制波よりも自由波に対してより不安定になるとき、次のような最終定常状態が出現する。すなわち、一定の振幅と位相速度をもつ自由波（傾圧波）が臨界安定な帯状流とつり合い、強制波は減衰してしまう。他方、帯状流が地形の影響をうける波に対してより不安定になるとき、両方の波が存在する。

この論文中で扱う最も自由度が大きい場合は、南北に2つのモード、東西に帯状成分と$A_2, B_1, A_1$の3つの波を許した28次元のモデルである。このとき、すべての波成分は地形と相互作用して、流れは時間とともに変化してゆく。変化の様子は次の4つの型に分けられ、その選択は、外部加熱による強制、摩擦、静的安定度といった外的パラメータに依存している：(1)一定の強制波と移動波を含む定常流、(2)周期振動流、(3)準周期的振動流、(4)不規則変動流。

地球大気に対応する外部パラメータを与えると、大きな振幅の波成分を含む不規則変動が出現する。この不規則変動をする状態中での帯状流と波動の関係を、長期間にわたる統計として調べた。各時刻での流れを、帯状流の強さを基準にして3つのカテゴリーに分類した。カテゴリー毎に合成した流れの場は次のような特徴をもつ：帯状流の卓越する高示数状態および中間状態。流れの線形する低示数状態である。この低示数状態では、帯状流の強さおよび鉛直シワが小さく、他の状態より停滞波の振幅が大きくなり、移動波の振幅が減少する。

不規則変動しているなかで、ある期間の平均として定義された停滞波は、第Ⅰ部で得られた平衡解の強制波と異った振幅・構造をしている。