A NUMERICAL EXPERIMENT ON THE EVOLUTION OF A POLAR VORTEX

SHIGEO YODEN and KEICHI ISHIOKA

Department of Geophysics
Kyoto University
Kyoto 606-01
Japan

ABSTRACT. In order to get a deeper insight into the stratospheric circulation during sudden warming events, evolution of a polar vortex perturbed by forced Rossby waves is investigated for a wide range of external parameters by using a high-resolution barotropic model similar to that used by Juckes and McIntyre (1987). For large amplitude of the wave forcing the polar vortex breaks down and the absolute vorticity is mixed irreversibly over the hemisphere. For small amplitude of the forcing, on the other hand, the vortex migrates off the pole during the forcing period and returns to the pole afterwards. Zonal wavenumber 2 forcing is more effective than wavenumber 1 to break the polar vortex. If the vortex is intensive, there is a clear separation between the reversible and the irreversible cases at a critical amplitude of the wave forcing. The evolution is highly dependent on the latitudinal profile of the initial mean zonal wind; if the mean zonal wind is symmetric with respect to the equator, the vortex breaks easily, while it is very robust for anti-symmetric mean zonal winds.

1. Introduction

McIntyre and Palmer (1983, 1984) introduced a new paradigm of breaking planetary waves and erosion of the circumpolar vortex in the winter stratosphere using daily maps of Ertel’s potential vorticity (PV) on the 850 K isentropic surface. The PV map is a powerful tool to diagnose the wintertime stratospheric circulation including stratospheric sudden warming events. Observational support for the paradigm has been done with several kinds of data and analyses (Leovy et al. 1985; Clough et al. 1985; Butchart and Remsberg 1986; Baldwin and Holton 1988). It has been widely recognized that the polar vortex as an isolated material entity and irreversible mixing outside the vortex due to breaking waves are intimately related to the variations of minor constituents such as ozone (McIntyre 1989; Schoeberl and Hartmann 1991).

Juckes and McIntyre (1987, hereafter referred to as JM) made a numerical experiment on the breaking planetary waves with a high-resolution one-layer hemispheric model. A prescribed polar vortex similar to the real winter stratosphere was perturbed by a time-dependent, quasi-topographic wave forcing of zonal wavenumber 1. Their result showed a clear evidence of breaking planetary waves and isolation of fluid inside the vortex from surroundings. Further experiments on the dynamics of the polar vortex have been done with similar two-dimensional models (Juckes 1989; Salby et al. 1990a, 1990b, 1990c; O'Sullivan and Salby 1990) and three-dimensional models (Kouker and Brassieur 1986; Mahlman and Umscheid 1987; O'Neill and Pope 1988; Haynes 1990).

Most of these numerical studies were limited to some particular cases close to the real strato-
Figure 1: (a) Latitudinal profile of initial absolute vorticity $Q_0(\phi)$. Dash-dotted line is the planetary vorticity $2\Omega \sin \phi$. (b) Zonal wind $u_0(\phi)$ corresponding to $Q_0(\phi)$.

sphere, because their main subjects were to investigate the time-dependent behavior of the flow field in detail, such as transport properties of the realized flow, formation of sharp PV gradients at the edge of the vortex, and so on. Details on the sensitivity of the time-dependent behavior to experimental parameters have not been reported yet as far as we know. In this study, several series of numerical experiments are done for a wide range of external parameters following JM. Sensitivity to the parameters, such as amplitude of the wave forcing, wavenumber of the forcing, intensity of the polar vortex and latitudinal structure of the vortex, is investigated with a primitive measure of the flow field. Recovery process of the polar vortex is also investigated in addition to breakdown process.

2. Model and experiment

We use a nondivergent barotropic vorticity equation similar to that introduced by JM:

$$\frac{DQ}{Dt} = (-\alpha + \nu \Delta^3)(Q - Q_0), \quad (1)$$

where $t$ is time and $D/Dt$ the material derivative. The dependent variable $Q$ is defined as

$$Q(\lambda, \phi, t) = \zeta_a(\lambda, \phi, t) + F(\lambda, \phi, t), \quad (2)$$

where $\zeta_a$ is the absolute vorticity and $F$ is a prescribed, quasi-topographic forcing function. Spherical coordinates $(\lambda, \phi)$ are longitude and latitude, respectively. Two kinds of damping terms are introduced in $(1)$: $\alpha$ is a Newtonian “cooling” coefficient and $\nu$ a coefficient of hyperviscosity for smooth numerical behavior. $\Delta$ is the horizontal Laplacian operator and $Q_0(\phi)$ is a prescribed, zonally symmetric equilibrium state with $F = 0$.

Equation $(1)$ is numerically integrated from an initial state $Q = Q_0$. Some initial profiles of $Q_0(\phi)$ used in this study are shown in Fig.1 together with the corresponding zonal wind $u_0(\phi)$. The streamfunction for two profiles denoted by solid line and dashed line is given by a single Legendre
polynomial $P_3(\sin \phi)$ and $P_2(\sin \phi)$, respectively. The prescribed, time-dependent wave forcing $F(\lambda, \phi, t)$ is given by the following form (see Juckes 1989):

$$F(\lambda, \phi, t) = 2\Omega \times F_0 A(t) B(\phi) \cos m\lambda.$$  

The time dependence $A(t)$ is assumed as

$$A(t) = \begin{cases} 
0.5(1 - \cos(\pi t/4)), & 0 \leq t \leq 4, \\
1, & 4 \leq t \leq 8, \\
0.5(1 + \cos(\pi(t - 8)/4)), & 8 \leq t \leq 12, \\
0, & 12 \leq t,
\end{cases}$$  

where the unit of time is day. The latitudinal dependence $B(\phi)$ is

$$B(\phi) = \begin{cases} 
\frac{\cot^2 \phi}{\cot^2 \phi_1} \exp(1 - \frac{\cot^2 \phi}{\cot^2 \phi_1}), & \phi \geq 0, \\
0, & \phi \leq 0,
\end{cases}$$  

where $\phi_1$ is the latitude at which $B(\phi_1) = \max B(\phi) = 1$. In this study the maximum of the wave forcing is placed at $\phi_1 = 60^\circ$. Therefore control parameters of the wave forcing in the present experiment are amplitude $F_0$ and zonal wavenumber $m$.

The Newtonian cooling coefficient $\alpha$ is set to be zero in most of the experiments, so that $Q$ is substantially a Lagrangian tracer except for the small hyperviscosity. The hyperviscosity coefficient used in this study gives a dissipation time-scale of 0.1 day at the largest total wavenumber $N$. A non-zero value of $\alpha = 0.1 \text{(day)}^{-1}$ is adopted in a series of experiments to see the relative importance of the diabatic process to the dynamic process in the recovery period of the polar vortex.

A pseudospectra method with a triangular truncation of $N = 85$ is used for the computation of the advection term. Grids for the spectral transformation are 256 (longitude) $\times$ 128 (latitude). The Runge-Kutta-Gill method is used for time integrations with $\Delta t = 0.01$ day. All of the computations are done in double precision.

There are some differences between the present model and that of JM: The present model is a full spherical model and has no “wall” at the equator. Spatial resolution of T85 is not so high as their T159, although both resolutions are very high compared with traditional stratospheric models. Some of our experiments have similar latitudinal profile of $Q_0(\phi)$ as theirs but others take different profile. In Eq. (4), $A(t) = 1$ for 4 days; this period is a half of theirs.

3. Results

3.1. EXPERIMENTS WITH NO NEWTONIAN HEATING

Several series of experiments with $\alpha = 0$ are done to see the dynamical process during the vortex erosion or breakdown. Figure 2 shows an example of the evolution of $Q$ field for the case similar to JM; the initial zonal wind is westerly in high and middle latitudes while easterly in low latitudes (solid line in Fig.1) and the amplitude of the $m = 1$ wave forcing is identical to the value taken by JM, $F_0 = 0.3$. The evolution resembles their result of Figs.3~5, particularly until day 6. A typical planetary-wave breaking process is observed: Fluid particles are well mixed outside the polar vortex and then latitudinal gradient of $Q$ is reduced there. The polar vortex remains as an isolated material entity with sharp gradients of $Q$ at the edge of the vortex. Deformation of the polar vortex by planetary-scale waves is still significant after the wave forcing was reduced to zero.

A time-latitude section of the mean zonal wind $\overline{u}$ is shown in Fig.3(a) for the same case of Fig.2. The westerly jet around $\phi = 60^\circ$ decelerates largely and the latitude of maximum $\overline{u}$ shifts poleward. After decreasing the wave forcing, the westerly jet recovers at a higher latitude with a narrower
Figure 2: $Q$ field at the times indicated for the $m = 1$ wave forcing of $F_0 = 0.3$. Contour value is scaled by $\Omega$ and dark shading corresponds to high $Q$. Lambert equal area projection is used only for a hemisphere.

Figure 3: (a) Time-latitude section of the zonal mean zonal wind $\overline{u}$. Contour interval is 20 m/s and dotted lines are for negative values. (b) Time variation of the mean zonal wind at $\phi = 60^\circ$ for five values of $F_0$ (0.1, 0.2, 0.3, 0.4, 0.5).
width. Easterly wind in low latitudes is intensified by the wave forcing. A hint of time-variation is also seen in the other hemisphere where in situ wave forcing is absent.

Sensitivity to the amplitude of the wave forcing is investigated for several values of $F_0$. Figure 3(b) shows the sensitivity of the initial polar vortex same as in Fig.2(a). As a crude measure of the evolution of the circumpolar vortex, the mean zonal wind at a latitude $\phi = 60^\circ$ is used in this study in addition to daily $Q$ maps. The mean zonal wind is a primitive measure but very familiar one in the study of stratospheric sudden warmings. For small amplitude of $F_0 = 0.1$ and 0.2, the mean zonal wind is perturbed little. As already shown in Fig.3(a) for $F_0 = 0.3$, it decelerates until day 9 but rapidly recovers to strong westerly wind after that time. For $F_0 = 0.4$ and 0.5, on the other hand, it changes to easterly wind during the forcing period and never recovers to westerly wind.

The $Q$ fields for these five cases at day 24 are shown in Fig.4 together with the initial field. For $F_0 = 0.1, 0.2$ and 0.3, high $Q$ area of the main polar vortex is nearly conserved for the integration period. Sharp gradients of $Q$ are formed at the edge of the vortex, which are located in lower latitudes for smaller $F_0$. Fluid particles outside the polar vortex are irreversibly mixed. Thus the quantity $Q$ is not conserved in the area of the irreversible mixing, because the hyperviscosity term is not negligible there owing to strong strain fields. On the other hand, the polar vortex is largely eroded for $F_0 = 0.4$ and 0.5; particularly for the latter case most of the polar fluids are shed out and mixed well with surroundings.

Sensitivity to the amplitude of wave forcing is studied also for the zonal wavenumber $m = 2$ with the same initial condition of $Q_0$ (figures are not shown). The $m = 2$ forcing is more effective than $m = 1$ to break the polar vortex if the amplitude of the forcing is the same. The mean zonal
wind at $\phi = 60^\circ$ is reduced to 20 m/s for $F_0 = 0.2$ and it changes to easterly wind for $F_0 \geq 0.3$. The high $Q$ area is split into two areas during the wave-forcing period even if the amplitude is small.

Dependency of the evolution on the intensity of the initial polar vortex is studied by changing the initial value $\pm 25\%$ with the same $P_3(\sin \phi)$ profile of the streamfunction. Figure 5 shows the time variation of the mean zonal wind at $\phi = 60^\circ$ for the weak-vortex case (a) and the strong-vortex case (b). For the weak-vortex case, the mean zonal wind recovers to westerly even for the largest wave forcing of $F_0 = 0.5$. On the other hand, the time variations are classified into two groups for the strong-vortex case; the mean zonal wind almost recovers to the initial value for $F_0 = 0.1, 0.2$ and 0.3, while it changes to strong easterly wind for $F_0 = 0.4$ and 0.5. The critical amplitude of the wave forcing that separates the two groups is $F_0 \sim 0.36$. The $Q$ fields for the strong-vortex case at day 24 are shown in Fig.6. For $F_0 = 0.1, 0.2$ and 0.3, the high $Q$ area in the polar region is not eroded very much and strong mixing takes place in low latitudes. For large wave forcing, on the other hand, the polar vortex breaks into two ($F_0 = 0.4$) or three ($F_0 = 0.5$) vortices, which keep their coherent structure as an isolated material entity for all the integration period. Note that the dispersion effect of Rossby waves is weak in these cases because latitudinal gradient of $Q$ becomes small in the hemisphere as a result of strong mixing.

Similar experiments were done for some different latitudinal profiles of the initial zonal wind. Figure 7(a) is a result for the profile denoted by a dashed line in Fig.4, the streamfunction of which consists of a single Legendre polynomial of $P_2(\sin \phi)$. Maximum value of the initial zonal wind is the same as the symmetric zonal wind denoted by a solid line but it is located at $\phi = 45^\circ$. The response is quite different from that for the symmetric zonal wind (Fig.3(b)). The mean zonal wind almost recovers to the initial value for all the cases of $F_0 = 0.1 \sim 0.5$. The polar vortex is very robust for the profile of initial zonal wind that is anti-symmetric with respect to the equator. The robustness is a common feature to other profiles of anti-symmetric zonal wind such as $\psi_0(\phi) = -|\sin^3 \phi|$. The initial state is largely different between the symmetric and anti-symmetric zonal winds in the other hemisphere where the wave forcing is absent. To investigate this sensitivity further, another series of experiments are done with the initial zonal wind denoted by a dotted line in Fig.4; the initial state for $\phi \geq 0^\circ$ is identical to that for the symmetric zonal wind and the difference is only in the other hemisphere. As shown in Fig.7(b), the mean zonal wind at $\phi = 60^\circ$ recovers to westerly even if amplitude of the wave forcing is large. The present result is compared with the symmetric case shown in Fig.3(b); the response is sensitive to the initial zonal wind in the other hemisphere particularly for the cases of large wave-forcing.
Figure 6: Same as Fig.4, but for the strong-vortex case. The initial value of $Q_0(\phi)$ is increased to $5/4$ of the standard value.

Figure 7: Same as Fig.3(b), but for the initial state indicated by dashed line in Fig.1 (a) and for the initial state indicated by dotted line (b).
3.2. EXPERIMENTS WITH NEWTONIAN HEATING

In order to investigate recovery process of the polar vortex after sudden warming events, a series of experiments were done with a Newtonian cooling coefficient of $\alpha = 0.1 \text{(day)}^{-1}$. Other experimental parameters are identical to those in the first series of experiments; the symmetric initial zonal wind with the standard intensity and the $m = 1$ wave-forcing with $F_0 = 0.1 \sim 0.5$.

Figure 8 shows the time-variations of mean zonal wind at $\phi = 60^\circ$, which have little difference from those in Fig.3(b) until day 4. However, the diabatic effect is evident after day 4 particularly in the cases of large breakdown of the vortex ($F_0 = 0.4$ and 0.5); the mean zonal wind recovers nearly exponentially after ceasing of the wave forcing. Thin solid lines in the figure give exponential relaxation to the initial prescribed value due to the Newtonian cooling since the mean zonal wind at $\phi = 60^\circ$ has a minimum value. Note that the recovery of the mean zonal wind is much faster than the diabatic process for $F_0 = 0.3$. In this case the dynamical process of vortex migration is dominant for the recovery; the circumpolar vortex, which migrated off the pole owing to the wave forcing, returns to the pole without much erosion of the main vortex.

4. Discussion

One of the most interesting results in this study is the sensitivity to the initial zonal wind profile in the other hemisphere where the wave forcing is absent. Difference between Fig.3(b) and Fig.7(b) is solely due to this sensitivity, because the experimental conditions are exactly the same in the hemisphere where the wave forcing exists. The sensitivity raises a question on the limitation of the theory of nonlinear critical layer when it is applied to the present experiments. The wave perturbation that is observed in the other hemisphere for symmetric mean zonal winds has a role in
the evolution of the polar vortex. It is conjectured that some quantity of the forced wave survives in the low-latitude easterly winds and reflects at the other pole to influence the critical layer interaction. Data analysis of these numerical results from the viewpoint of nonlinear interactions of latitudinally propagating planetary waves is an interesting subject in the near future.

Mean zonal wind at a latitude ($\phi = 60^\circ$) was used as a primitive measure of the evolution of the polar vortex. The measure is a familiar one in the study of stratospheric sudden warmings but we need more appropriate measure with a dynamical basis to characterize the evolution of the polar vortex. For this purpose McIntyre and Palmer (1983, 1984) introduced an area index of the main polar vortex, which was used in the analysis of the LIMS data by Butchart and Remsberg (1986). It is interesting to see the usefulness of the area index with the data obtained in the present experiments. Dynamical theory on the nonlinear evolution of a vortex (or some vortices) on a rotating sphere has not been developed satisfactorily until now compared with the development of the wave theory on a rotating sphere. It is an important subject to establish the dynamical theory of vortex evolution.

5. Conclusion

Several series of numerical experiments on the evolution of a polar vortex are done with a high-resolution barotropic model in a spherical domain in order to get a deeper insight into the evolution of the stratospheric circulation during sudden warming events. A prescribed westerly circumpolar vortex is perturbed by forced Rossby waves following the pioneering work by JM, and sensitivity of the evolution to several experimental parameters is investigated, such as amplitude of the wave forcing, wavenumber of the forcing, intensity of the vortex, and latitudinal profile of the vortex.

For large amplitude of the wave forcing the polar vortex breaks down and the absolute vorticity, which is a Lagrangian tracer after ceasing of the wave forcing, is mixed irreversibly over the hemisphere. For small amplitude of the forcing, on the other hand, the polar vortex migrates off the pole during the forcing period and returns to the pole reversibly. Zonal wavenumber 2 forcing is more effective than wavenumber 1 to break the polar vortex if the amplitude of the forcing is the same. In a series of experiments with strong initial vortex, there is a clear separation between the reversible and the irreversible evolutions at a critical amplitude of the wave forcing, namely the response sharply depends on the amplitude around the critical value. The evolution is highly dependent on the latitudinal profile of the initial mean zonal wind; if the mean zonal wind is symmetric with respect to the equator, the vortex breaks easily, while it is very robust for anti-symmetric profiles of the mean zonal wind.

Recovery process of the polar vortex after ceasing of the wave forcing was also investigated in the experiments with a Newtonian cooling term. Relative importance of the diabatic process to the dynamic process depends on the degree of breakdown of the vortex. If the vortex has only reversible deformation, the dynamical process is dominant and the circumpolar vortex recovers rapidly; the time-scale is much shorter than that of diabatic relaxation. On the other hand, the polar vortex recovers slowly by the diabatic process when the vortex breaks down largely. Dynamical process such as vortex merger is not so effective at least in our experiments.

References


