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Atmospheric Predictability

Shigeo YODEN

Department of Geophysics, Kyoto University, Kyoto, Japan

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Abstract

The histories of numerical weather prediction and atmospheric predictability research are briefly reviewed in this article in celebration of the 125-year anniversary of the foundation of the Japan Meteorological Society. The development of numerical weather prediction in the 20th century has been intimately related to the progress of dynamic meteorology as stated in Section 1, including the development of the quasi-geostrophic system that is a basic tool to describe large-scale balanced flow approximately and the discovery of chaos that is the key concept of atmospheric predictability. In the 1990s, the rapid advancement of computer technology brought a regime shift in the predictability research from fundamental theoretical works with simple nonlinear dynamical systems (Section 2) to practical applied works with operational numerical weather prediction models (Section 3). Ensemble forecasts became in operations in major forecast centers at the end of the 20th century. Some current challenges in the atmospheric predictability research under THORPEX (THe Observing system Research and Predictability EXperiment) program are summarized in Section 4, such as targeted observations, new data-assimilation techniques, and interactive grand global ensemble forecasts.

1. A historical perspective

a. *Fledgling period of numerical weather prediction*

Historical review of numerical weather prediction (NWP) and atmospheric predictability research could be started by referring to the paper published over a century ago by Bjerknes (1904). He considered the problem of weather prediction from the standpoint of mechanics and physics and proposed it as a deterministic initial value problem based on the physical laws such as the conservation of mass, momentum, and energy. The first trial of NWP was done by Richardson (1922) employing a finite difference method, not graphical methods, to deal with the governing partial differential equations. His unrealistic prediction of surface pressure change of 145 hPa over 6 hours after enormous amount of calculations by hand is now attributable to imbalance in the initial data used by him but not to his method (Lynch 1999).

A quarter of a century later, "Richardson's dream" of NWP came true by the development of electronic digital computer that provided a means of processing enormous amount of calculations within affordable time span (e.g., Thompson 1983; Wiin-Nielsen 1991; Cressman 1996; Lynch 2002). In the late 1940s, Electronic Numerical Integrator And Computer (ENIAC) was developed at Princeton's Institute for Advanced Study (IAS), and it was used for attacking the problem of NWP by the Meteorological Research Group (Platzman 1979). Charney et al. (1950) succeeded in 24-hour forecasts computed from actual data at the 500 hPa level over and around North America with a simplified barotropic vorticity equation model.

In the fledgling period of NWP, there were

Corresponding author: Shigeo Yoden, Department of Geophysics, Graduate School of Science, Kyoto University, Kitashirakawa Oiwake-cho, Sakyo-ku, Kyoto 606-8502, Japan.
E-mail: yoden@kugi.kyoto-u.ac.jp

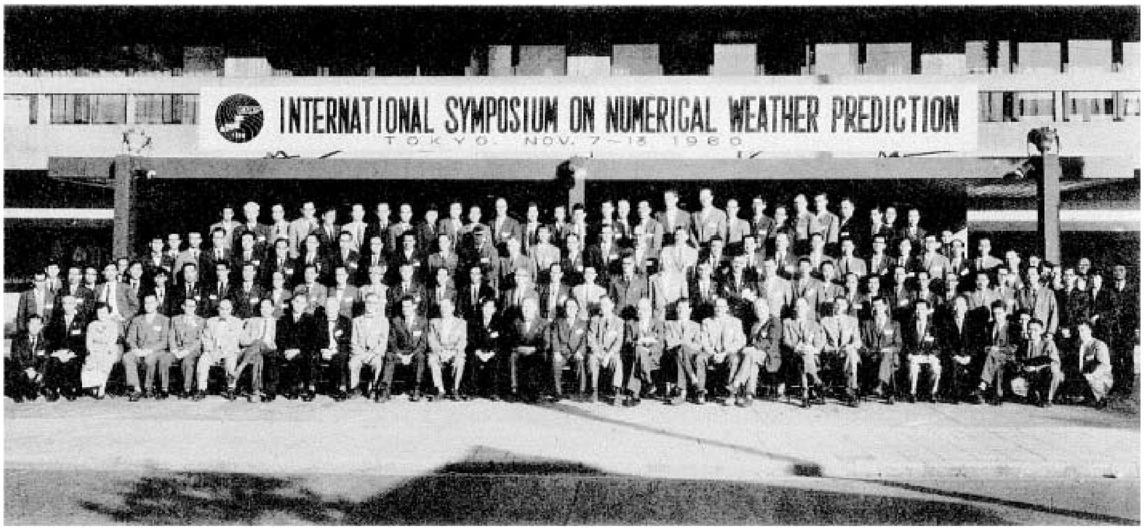


Fig. 1. Group photo of the first International Symposium on Numerical Weather Prediction held in Tokyo on 7–13 November 1960 (Syono, 1962).

significant associations and contributions from the meteorological research community in Japan. In Jule Charney's recollections on the period of ENIAC (Platzman 1990, page 53), one can find a description on the visits of the director general, Kiyoo Wadati of the Japan Meteorological Agency (JMA), and Professor of Meteorology, Shigekata Shono of University of Tokyo. Kanzaburo Gambo, who succeeded Professor Syono in the 1970s, was invited to work at the IAS as a research associate from 1952 to 1954 (Lewis 1993). After his return to Japan with infectious excitement about the challenges to NWP, Syono and Gambo coordinated the research group of NWP in Tokyo. JMA started operational NWP in 1959, after the first real-time operational NWP in Sweden in 1954 and the second in the US in 1955 (Kalnay 2003).

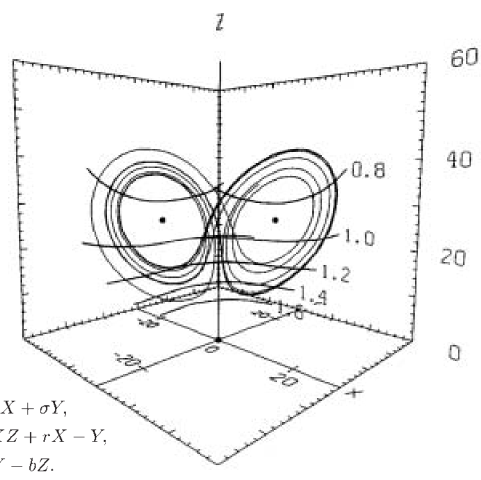
A milestone in that period was the first international symposium on NWP held in Tokyo on 7–13 November 1960 (Fig. 1; Syono 1962). There were over 50 participants from abroad in the field of dynamic meteorology and NWP, including Jule G. Charney, Arnt Eliassen, Ragner Fjørtoft, Hsiao-Lan Kuo, Edward N. Lorenz, Yale Mintz, Jerome Namias, Norman A. Phillips, George W. Platzman, Joseph Smagorinsky and others. "Looking back, it is fair to say that the meeting was one of the epoch-making events in the history of NWP", as Akira Kasahara said (Lewis 1993). The international symposium provided the opportunity for "talented young scientists in Japan to meet in person many of the leading scientists who attended", and consequently it was an epoch-making event which facilitated exodus of Japanese meteorologists to the United States in those days; there were presentations by Akio Arakawa, Tetsuya Fujita, Akira Kasahara, Yoshio Kurihara, Kikuro Miyakoda, Yoshimitsu Ogura, Yoshikazu Sasaki, and Michio Yanai, some of whom attended from the United States.

b. *Lorenz's discovery of chaos*

At the international symposium in Tokyo, Edward Lorenz presented a paper titled "The statistical prediction of solutions of dynamical equations" (Lorenz 1962, 1993). He demonstrated that statistical predictions based on linear regression methods become inadequate beyond a few days, by applying the methods to irregular nonperiodic solutions obtained numerically in a set of 12-variable ordinary differential equations (ODEs) that was a highly truncated spectral model of a two-layer baroclinic fluid. It is interesting to note the discussion on the exponential growth of a small initial error in the nonperiodic solutions to answer the question, "Did you change the initial condition just slightly and see how much different results were in the forecasting in this way?" asked by Bert Bolin. Lorenz's presentation in Tokyo was a prelude to his most famous and influential paper published in 1963.

By using a "personal" computer Royal-McBee LGP-30, Lorenz (1963) investigated nonperiodic solutions in the minimum system of 3-variable nonlinear ODEs derived from the equations on thermal convection. He found that the nonperiodic solutions are unstable so that slightly differing initial states can evolve into considerably different states within a limited time. This sensitivity to initial conditions is a fundamental nature of chaos in later terminology. Sometimes chaos is regarded one of the three great revolutions in the 20th-century physical sciences in addition to relativity and quantum mechanics, because chaos cut away at the tenets of Newton's physics and eliminated the Laplacian fantasy of deterministic predictability (Gleick 1987).

Lorenz (1963) illustrated a chaotic solution obtained by numerical time integrations as a trajectory in 3-dimensional phase space, which is now well known as the Lorenz attractor (Fig. 2). Such illustrations of the Lorenz attractor can be found popularly in textbooks on chaos, nonlinear science, and predictability (e.g., Sparrow 1982; Drazin 1992; Glendinning 1994; Sprott 2003; Palmer and Hagedorn 2006). Actually, it was the early 1970s that James Yorke, a distinguished applied mathematician who gave the science of chaos its name, discovered Lorenz (Li and Yorke 1975; Gleick 1987). Since then, Lorenz's pioneering work published in 1963 in the *Journal of the Atmospheric Sciences* became well known in a wider community of chaos and nonlinear science. Edward Lorenz received the 1991 Kyoto Prize[1] for his establishment of the theoretical basis of weather and climate predictability, as well as the basis for computer-aided atmospheric physics and meteorology. At the commemoration workshop titled "From Weather Forecasting to Chaos", I had a chance to make a presentation on "Local Lyapunov stability and atmospheric predictability" (Yoden and Nomura 1993), which was largely dependent on his theoretical works on atmospheric predictability as summarized in Section 2.



$$\begin{cases} \dot{X} = -\sigma X + \sigma Y, \\ \dot{Y} = -XZ + rX - Y, \\ \dot{Z} = XY - bZ. \end{cases}$$

Fig. 2. Trajectory of the Lorenz attractor for the standard parameters of $\sigma = 10$, $r = 28$ and $b = 8/3$ in a three-dimensional perspective and the distribution of the Lorenz index defined by Eq. (6) for the optimization interval of $\tau = t_1 - t_0 = 0.1$ (Mukougawa et al. 1991).

c. *Chaos, turbulence, and predictability theories*

In 1983 three international conferences on chaotic phenomena, turbulence, and predictability in fluids were held in North America, Europe, and Asia: American Institute of Physics Conference at La Jolla Institute, U.S. in February (Holloway and West 1984), International School of Physics 《Enrico Fermi》 Course at Varenna on Lake Como, Italy in June (Ghil et al. 1985), and International Symposium of the International Union of Theoretical and Applied Mechanics in Kyoto, Japan, in September (Tatsumi 1984). The last one was the first international meeting that I participated in (Yoden and Hirota 1984).

Turbulence is a disordered flow characterized by complicated and irregular fluid motions in time and space. Traditionally statistical approaches had been taken in theoretical studies on turbulence with stochastic tools, although fluid turbulence is a deterministic phenomenon governed by the Navier-Stokes equations. Even for a deterministic system, unpredictability and randomness is introduced by the nonlinearities

1 http://www.inamori-f.or.jp/laureates/k07_b_edward/prf_e.html

of the equations and an extremely large number of spatial degrees of freedom which is needed to describe the motion (e.g., Lesieur 1997). From the 1970s to 80s, the development of chaos theories in nonlinear dynamical systems had introduced a new approach to the problem on the onset of turbulence.

Hydrodynamic instabilities and turbulence had been extensively studied for more than a century, while in the 1970s–80s the situation of their research was changed dramatically by the use of computers in numerical analyses of nonlinear systems and in laboratory experiments (e.g., Swinney and Gollub 1981). Chaos theory, or dynamical systems theory, had clarified the problem of the onset of turbulence (e.g, Ruelle and Takens 1971; Read 2001). The characteristic time that determines sensitivity to initial conditions should decrease as the number of degrees of freedom of system increases, and it is so short in fully developed turbulence with a large number of degrees of freedom that unpredictability and randomness are introduced even for a deterministic system.

A change in the small-scale structure in turbulence will, in due time, produce a change in the large-scale structure, and we can expect that in a few hours or a day the imperceptible manipulation of a little devil has resulted in a change of the atmospheric turbulence on a scale of kilometers (Ruelle 1991). Beyond that, large-scale atmospheric motions are not fully developed turbulence, but coherent structures exist, such as fronts, tropical and extratropical cyclones, blocking highs, jet streams, planetary-scale waves, and so on. The time needed for the amplification of a small-scale observational error to a globally different situation is estimated to be one or two weeks. The successes and failures of NWP in extended range are the empirical basis of predictability theory (Ghil et al. 1985).

d. Advancement of NWP

In the 1960s and 70s, sensitivity to the initial conditions had been argued in the predictability theories under perfect-model assumption. In those days, however, NWP models were far from perfect, and much effort had been made to improve the models. There were many important contributions by Japanese scientists to the improvements, including novel and stable finite-difference methods developed in the 1960s, such as the Kurihara (1965) grid, the Arakawa (1966) Jacobian and the Matsuno (1966) scheme, and sophisticated parameterization schemes of sub-grid scale phenomena developed in the 1970s, such as the Arakawa and Schubert (1974) cumulus parameterization and the Mellor and Yamada (1974) turbulence closure models for planetary boundary layers.

In addition to the improvements of NWP models, the advancement of computer technology, such as computational speed and memory size, has continued from the early era. Computational speed has an exponential growth from 3×10^2 FLOPS (FLoating point number Operations Per Second) of ENIAC to 10^{13} FLOPS over the last half century. Now the Earth Simulator[1] in Yokohama, which was the fastest supercomputer in the world from 2002 to 2004, has the total peak performance of 40 TFLOPS and the total main memory of 10 TB.

Figure 3 shows the evolution of mean forecast skill at the European Centre for Medium-Range Weather Forecasts (ECMWF) for the northern and southern hemispheres for the period of 1981–2004 (Shapiro and Thorpe 2004). Forecast skill is judged with some measures of forecast errors defined by the difference between the forecast and analysis for a given verification time. Here "analysis" means an initial condition estimated from observed data. The forecast errors are caused by two factors: (1) the imperfection of the model and (2) the growth of errors included in the initial conditions. The latter is the main concern of the chaos theory. Continual improvements in medium-range weather forecasts are due not only to the improvements of NWP models and the advancement of computer technology, mainly associated with the reduction of the factor (1), but to the increase of observation data over the globe, including satellite measurements, and the development of advanced techniques of data assimilation and ensemble forecast, mainly associated with the reduction of the factor (2).

e. Forecast skill variations and their prediction

As the medium-range forecasts had become skillful, growth of errors included in the initial

1 http://www.es.jamstec.go.jp/esc/eng/index.html

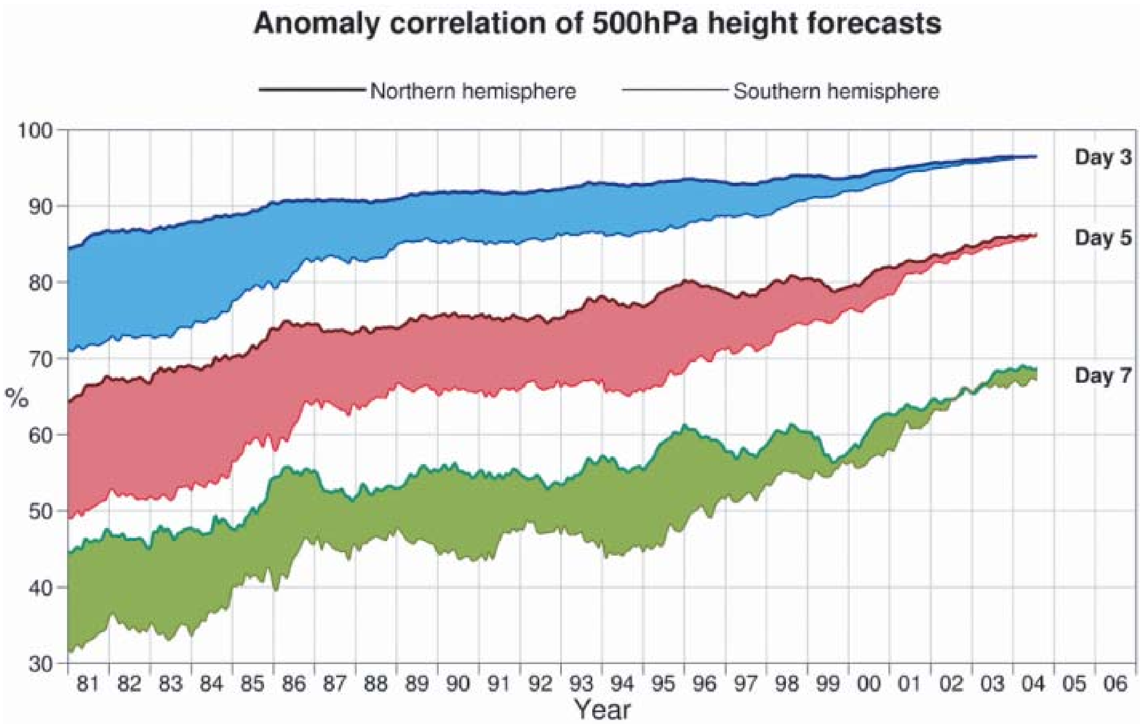


Fig. 3. Evolution of mean forecast skill for the extratropical northern and southern hemispheres for the period of 1981–2004 (Shapiro and Thorpe 2004). Shading shows differences between hemispheres in anomaly correlation of 3, 5, and 7-day ECMWF 500-hPa height forecasts.

conditions by observations and analysis became a real problem in operational NWPs. Lorenz (1982) computed the growth of forecast error, i.e., the difference between forecast and analysis, for 10-day forecasts at ECMWF averaged over a 100-day period (thick curve in Fig. 4). He also computed the mean growth of the difference between two forecasts verifying at the same time, but started from initial conditions that are one ($k - j = 1$) day apart (the lowest thin curve), two day apart, and so on. The mean growth of small differences can be assumed as

$$\frac{dE}{dt} = aE - bE^2, \tag{1}$$

where E is an ensemble mean of small differences and t is time. The constant a measures the exponential growth rate of small differences, and the quadratic term halts the growth with $a > 0$ and $b > 0$ (i.e., nonlinear saturation of the error growth). Based on the estimate of the doubling time of small errors as 2.5 days with Eq. (1) and the data used for plotting Fig. 4, Lorenz (1982) concluded that forecasts of instantaneous weather patterns nearly two weeks in advance appear to be possible. Further arguments on the mean growth of global forecast errors can be found in Lorenz (1985) and Kalnay and Livezey (1985).

The growth of small initial errors varies in time and space because of the nature of flow-dependent predictability. It has been widely recognized that NWP models show large time variations in skill in medium-to-extended range. Figure 5 shows recent examples of one-month ensemble forecasts which show high sensitivity to the initial condition for the prediction of stratospheric sudden warming event in December 2001 (Mukougawa et al. 2005). There is a clear difference in the divergence of ensemble members among the three different forecast periods. Prolonged predictability is observed during the period of a stratospheric sudden warming event (Fig. 5c), while all members diverge rapidly during the onset period of the sudden warming event (Fig. 5b).

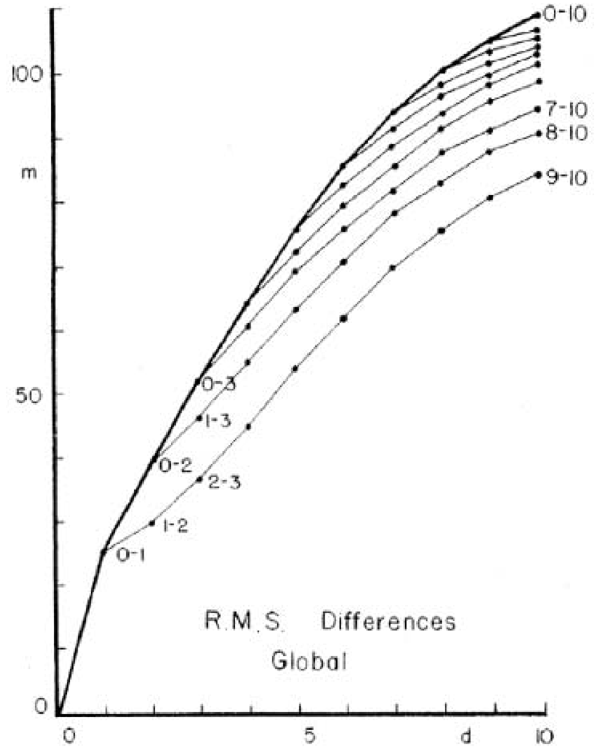


Fig. 4. Growth of forecast error for operational 10-day forecasts at ECMWF averaged over a 100-day period (Lorenz 1982). Global root-mean-square 500-hPa height differences [m] between j-day and k-day forecasts for the same verification day, for $j < k$, plotted against k. Some values of j-k are shown beside the points. Thick curve connects the difference of the k-day forecast and analysis ($j = 0$), while thin curves connects the spread between j-day and k-day forecasts.

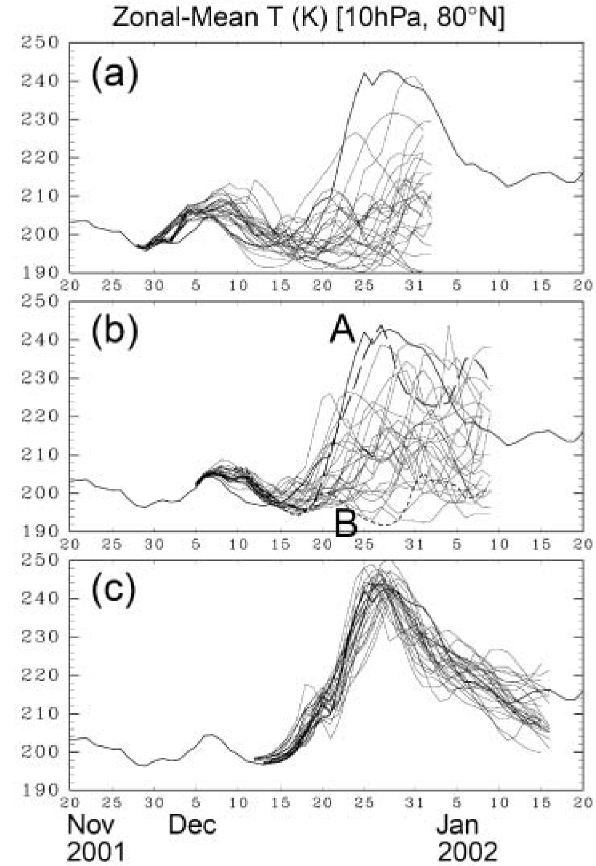


Fig. 5. Time variation of the spread of the JMA one-month ensemble forecasts for the period of a stratospheric sudden warming event in December 2001 (Mukougawa et al. 2005). Zonal mean temperature at 10-hPa and 80 N for the analysis is plotted from 20 November 2001 through 20 January 2002 (thick solid line) and those for ensemble forecasts (thin solid lines) from (a) 28 and 29 November, (b) 5 and 6 December, and (c) 12 and 13 December.

JMA started operational medium-range (8-day) forecast every day in March 1988, and Kimoto et al. (1992) investigated skill variations in the first winter. They compared skill variations of 7-day forecasts at JMA with those at ECMWF and at the U.S. National Meteorological Center (NMC; the direct precursor to the National Centers for Environmental Prediction, NCEP) and noticed correlated variations among all the different models (Fig. 6). They studied the common poor skills at the end of January and their association with a typical blocking phenomenon over the North Pacific (i.e., forecast divergence prior to the onset of blocking), and argued that the poor skills reflected temporal variations in atmospheric predictability, not common deficiencies for a particular circulation pattern.

Large time-variations in skill in medium-to-extended range NWPs raised a challenge of *forecasting* forecast skill. There had been some early trials on operational skill forecasting (e.g., Kalnay and Dalcher 1987; Palmer and Tibaldi 1988; and references therein). In those days, the lagged average forecast (LAF) method

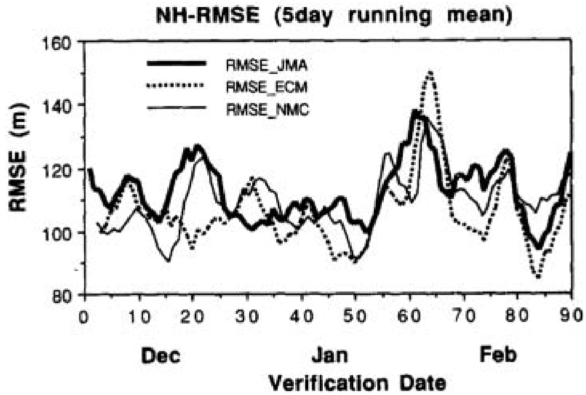


Fig. 6. Evolution of the root-mean-square errors (RMSEs) for the extratropical northern hemisphere (>20N) at three operational centers in the winter 1988–89 (Kimoto et al. 1992). RMSE of 7-day 500-hPa height forecasts for JMA (thick solid line), ECMWF (dotted line), and NMC (thin solid line) are plotted after applying 5-day running average. The abscissa is the verification date, day 1 being 1st in December 1988.

(Hoffman and Kalnay 1983) was an operationally feasible "ensemble forecast" method without extra computer resources. The LAF method provided *a priori* estimates of forecast skill because there was a strong correlation between the dispersion of the ensemble members and the loss of predictability. Kimoto et al. (1992) also proposed a new method to give a linear measure of forecast spread, in which a tangent linear equation of a hemispheric barotropic model was used to make a singular value analysis (see Section 2) with forecasted reference fields.

However, the advancement of computer technology was so fast that these methods became obsolete soon. The new era of operational ensemble forecasting has come in the early 1990s, although the prediction of predictability is still a theoretical challenge even now (e.g., Ziehmann et al. 2000).

f. Ensemble forecast as an application of chaos theory

The prediction problem of forecast uncertainty can be investigated theoretically in terms of the Liouville equation that governs the time evolution of the probability density function (PDF) of a state vector of a given dynamical model (Ehrendorfer 1994a, 1994b, 1997). Such an approach is, however, not practical in forecasting forecast skill in NWP with a model which has an extremely large number of degrees of freedom. A related concept more readily applicable to operational NWPs is stochastic dynamic prediction with some closure assumptions on the PDF, firstly proposed by Epstein (1969). A recent example of the impact of a random effect of probabilistic terms is the "stochastic parameterization" of unresolved processes that is related to the PDF of model error (e.g., Palmer 2001, 2002a). The stochastic parameterization is another way to represent model uncertainty in ensemble forecasts, compared with multi-parameter-value ensembles or multi-model ensembles which will be briefly mentioned in the next section.

Leith (1974) investigated the theoretical skill of Monte Carlo approximations to the stochastic dynamic prediction. In a Monte Carlo approach, a large ensemble of initial conditions is generated by random sampling of the initial PDF and time integrations from each initial condition are done to estimate the evolution of the PDF. In the operational NWP contexts, the initial condition should be given by a form with a PDF which represents small errors introduced by observations and analysis.

Random sampling of the initial PDF is not efficient in operational NWP contexts because of the large degrees of freedom of the NWP models. A sufficiently large sample size is necessary to obtain reliable estimates of the evolution of the PDF. In the early 1990's, the chaos theory that describes time evolutions of initial small perturbations in nonlinear dynamical systems had been applied to operational NWPs to generate an optimal set of initial conditions that represents the initial PDF (e.g., Mureau et al. 1993; Toth and Kalnay 1993). The sample size was set to the order of 10, a plausible number in operational NWPs, by using such methods as to generate dynamically conditioned perturbations. The term "ensemble forecast" is often used in a limited sense as such, and its concise summary is given in Section 3. A historical review on roots of ensemble forecast can also be found in Lewis (2005). Recent topics on predictability in weather and climate, including ensemble systems for forecasting predictability, are thoroughly reviewed in a book edited by Palmer and Hagedorn (2006).

Further challenges in atmospheric predictability research beyond the success of ensemble forecast are now taken under the international research program of THORPEX (THe Observing system Research and Predictability EXperiment) to accelerate improvements in the accuracy of one-day to two-week high impact weather forecasts for the benefit of society, the economy, and the environment (Shapiro and Thorpe 2004; Rogers et al. 2005; WMO 2005a). Some of the fundamental and important aspects of the THORPEX are described in Section 4.

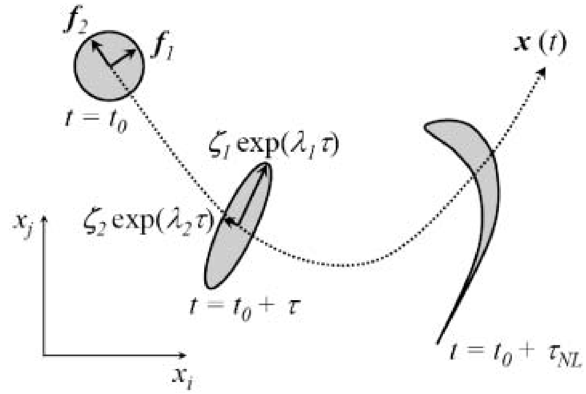


Fig. 7. Illustration of the evolutions of a reference solution $\boldsymbol{x}(t)$ and small perturbations superposed on it at an initial state t_0 projected onto x_i-x_j plane in phase space. See text for details.

2. Theories on atmospheric predictability

a. Lyapunov stability

Lorenz (1965) pioneered in constructing a fundamental framework of the growth of small errors superposed on a reference solution for a prescribed finite time interval. A general treatment of the concept for infinite time interval was independently provided by Oseledec (1968). The Oseledec theorem, which is well known in the community of chaos and nonlinear science, implies that nearby trajectories, separated initially by a small distance in phase space, will separate exponentially at a rate given by the Lyapunov exponent (see Section 2.*c*). It is a global property of attractor and can be used for classifying each attractor. A positive value of the Lyapunov exponent indicates an exponential loss of correlation between two nearby trajectories, which gives the mathematical definition of chaos. For further material on the Lyapunov stability, see textbooks on nonlinear dynamical systems, e.g., Guckenheimer and Holmes (1983).

Local or finite-time Lyapunov exponent was introduced in the same framework as Lorenz (1965) to study the local property of attractor (e.g., Goldhirsch et al. 1987; Abarbanel et al. 1991). The finite-time Lyapunov stability analysis has been applied to fully developed model turbulence (Ohkitani and Yamada 1989; Kida et al. 1990) and to atmospheric predictability (e.g., Lacarra and Talagrand 1988; Farrell 1990; Houtekamer 1991; Mukouga et al. 1991; Yoden and Nomura 1993; Molteni and Palmer 1993; Trevisan and Legnani 1995; Anderson 1996; Yamane and Yoden 1997, 2001; Trevisan and Pancotti 1998).

b. The growth of small errors: Lorenz (1965)

Let us consider a nonlinear dynamical system of dimension n:

$$\frac{d}{dt}\boldsymbol{x}(t) = \boldsymbol{f}[\boldsymbol{x}(t)],$$

$$\boldsymbol{x}(t) = [x_1(t), \ldots, x_n(t)]^T \in \boldsymbol{R}^n, \tag{2}$$

where $(\ )^T$ denotes transpose. A solution of the system, $\boldsymbol{x}(t)$, is uniquely determined by setting an initial state $\boldsymbol{x}(t_0)$ and is called a reference solution, which is given by a trajectory in phase space as illustrated in Fig. 7.

An infinitesimally small perturbation $\boldsymbol{y}(t)$ superposed on the solution $\boldsymbol{x}(t)$ obeys the tangent linear equation of Eq. (2):

$$\frac{d}{dt}\boldsymbol{y}(t) = \mathsf{J}[\boldsymbol{x}(t)]\boldsymbol{y}(t), \tag{3}$$

where $\mathbf{J}[\boldsymbol{x}(t)]$ is the Jacobian matrix, $\mathsf{J}[\boldsymbol{x}(t)] \equiv (\partial \boldsymbol{f}/\partial \boldsymbol{x})_{\boldsymbol{x}=\boldsymbol{x}(t)}$. The solution of Eq. (3) can be written in the form of a linear transformation with an $n \times n$ matrix $\mathbf{M}(t_1, t_0)$ as

$$\boldsymbol{y}(t_1) = \mathsf{M}(t_1, t_0)\boldsymbol{y}(t_0). \tag{4}$$

The matrix $\mathbf{M}(t_1, t_0)$ is referred to as the error matrix.

Lorenz (1965) described the linear evolution of isotropic random perturbations with the error matrix. An isosurface of the PDF of initial perturbations is an n-dimensional sphere with a radius ε as illustrated in Fig. 7. According to Eq. (4), this sphere is deformed into an ellipsoid at $t_1 = t_0 + \tau$, and the length and direction of semiaxes of the ellipsoid are given by the singular values and vectors of the transpose of the error matrix $\mathbf{M}^T(t_1, t_0)$ (Lorenz 1965; Yoden and Nomura 1993); if the symmetric matrix $\mathbf{M}(t_1, t_0)\ \mathbf{M}^T(t_1, t_0)$ has non-negative eigenvalues $\{\Gamma_i^2\}$ and eigenvectors $\{\boldsymbol{\zeta_i}\}$, the lengths of the semiaxes of the ellipsoid are $\{\varepsilon\Gamma_i\}$ and the directions of the semiaxes are $\{\boldsymbol{\zeta_i}\}$ (Fig. 7).

If we define the amplification rate of a perturbation $\boldsymbol{y}(t_0)$ from t_0 to t_1 as

$$\gamma(t_1, t_0, \boldsymbol{y}(t_0)) \equiv \frac{\|\mathbf{M}(t_1, t_0)\boldsymbol{y}(t_0)\|}{\|\boldsymbol{y}(t_0)\|}, \tag{5}$$

with a norm $\|\cdot\|$, then the root-mean-square amplification rate of the perturbations distributed equally in phase space at $t = t_0$ is given by the root-mean-square of singular values of the error matrix $\mathbf{M}(t_1, t_0)$ (Lorenz 1965). This quantity, denoted as $\alpha(t_1, t_0)$, has been referred as the Lorenz index and used as a standard measure of the perturbation growth:

$$\alpha(t_1, t_0) \equiv \sqrt{\frac{1}{n}\sum_{i=1}^{n}\Gamma_i} = \sqrt{\frac{\mathrm{tr}(\mathbf{M}\mathbf{M}^T)}{n}}. \tag{6}$$

c. *Lyapunov exponents and vectors*

If the singular values and vectors of the error matrix $\mathbf{M}(t_1, t_0)$ are denoted as $\{\exp[\lambda_i\tau]\}$ and $\{\boldsymbol{f}_i\}$, respectively, then the finite-time Lyapunov exponents $\{\lambda_i\}$ can be written as

$$\lambda_i(\boldsymbol{x}(t_0), \tau) = \frac{1}{\tau}\ \log\|\mathbf{M}(t_1, t_0)\boldsymbol{f}_i(\boldsymbol{x}(t_0), \tau)\|, \tag{7}$$

for an initial state $\boldsymbol{x}(t_0)$ and a finite time interval τ (Goldhirsch et al. 1987; Yoden and Nomura 1993). Nearby trajectories separated initially by a small distance in the direction of $\boldsymbol{f}_i$ will separate in time at a rate $\exp[\lambda_i t]$ as illustrated in Fig. 7. If we take a limit of $\tau \to \infty$ in Eq. (7), the limit becomes independent of the initial state to have a global property of the attractor. It is this limit of Eq. (7) that gives the definition of the Lyapunov exponents. Thorough explanation of the Lyapunov exponents and vectors and their finite-time version can be found in e.g., Legras and Vautard (1996) and Kalnay (2003), although there is some confusion about the terminology. The vectors $\{\boldsymbol{f}_i\}$ are called as the forward (right, initial, ...) singular vectors, or simply, the singular vectors (SVs), while $\{\boldsymbol{\zeta_i}\}$ are the backward (left, final, evolved, ...) singular vectors.

d. *A simple example of the variation of error growth*

We illustrate the time variation of error growth and the use of some measures defined above with a simple example of the Lorenz (1963) system of a 3-variable nonlinear dynamical system (a set of ODEs). A trajectory of the Lorenz attractor for the standard parameters of $\sigma = 10$, $r = 28$ and $b = 8/3$ is shown as a three-dimensional perspective in Fig. 2. The Lyapunov exponents for this example are computed[1] as $\lambda_i = 0.90563, 0,$ and -14.57219, with which the Kaplan-Yorke (Lyapunov) dimension of the attractor is obtained as $D_{KY} = 2.06215$ (Sprott 2003). Thus, the well known character of the Lorenz attractor is reconfirmed that it is a chaotic system ($\lambda_1 > 0$) with a fractal dimension of the attractor slightly greater than 2.

The finite-time Lyapunov exponents $\{\lambda_i(t_1, t_0)\}$ and the Lorenz index $\alpha(t_1, t_0)$ can be computed on the Lorenz attractor for a given optimization interval τ. Figure 2 also shows the distribution of the Lorenz index $\alpha(t_1, t_0)$ for a short time interval $\tau = 0.1$ (Mukougawa et al. 1991). The time variation of predictability can be recognized as phase-spatial organization of the Lorenz index; the error growth is large in the bottom part of the attractor that is close to the origin. More intuitive presentation of the local variation of predictability on the Lorenz attractor is given in Palmer (1993, Fig. 4). However, the Lorenz index does not always show distinct relationship with the unstable stationary points denoted by dots, contrary to the suggestion of a dynamical role of quasi-stationary states, which can be generated by the unstable stationary points, in the time variation of predictability (e.g., Legras and Ghil 1985). The Lorenz index increases monotonically during quasi-stationary states only in one-dimensional dynamical system, while there is no such

1 Lyapunov Exponent Spectrum Software by Sprott (2005) is found at http://sprott.physics.wisc.edu/chaos/lespec.htm

unique relationship in multi-dimensional systems (Yamane and Yoden 1997).

e. Application to systems with large dimension

If the system is not very large, we can obtain the error matrix directly by repeating the time integration of the nonlinear dynamical system (2) n-times from a new initial condition with a small perturbation $\boldsymbol{y}(t_0)$ which has non-zero value only in the i-th component for $1 < i < n$ (Lorenz 1965). Using this procedure to obtain the error matrix of $n = 1848$, Yamane and Yoden (2001) computed all singular values and vectors of the $n \times n$ matrix numerically for the finite-time Lyapunov stability analysis. Under current computing facilities, a larger system with $O(10^4)$ degrees of freedom could be analyzed by this straightforward method.

For operational NWP systems with $O(10^{6\sim7})$ degrees of freedom, however, it is neither practical nor meaningful to obtain *all* the singular values and vectors, because a large part of them could be associated with small-scale disturbances with small amplification rate for the optimization interval of a few days. Lacarra and Talagrand (1988) used the adjoint of the tangent linear equations (3) and iterative Lanczos algorithm for symmetric matrices to determine the largest singular value and vector in a barotropic model. Houtekamer (1991) estimated that 150 model runs are necessary to obtain 50 of the largest singular values and corresponding vectors with the adjoint method.

Mureau et al. (1993) firstly applied the adjoint method as a means of providing dynamically conditioned perturbations for ensemble forecasting at ECMWF (see also the next section). Figure 8 shows an example of the horizontal structure of the leading singular vector used in operational ensemble forecasting, taken from a review paper on ensemble forecasts by Buizza (2001). As in general cases (Buizza and Palmer 1995), the initial perturbation is localized in a region of storm track cyclogenesis with most amplitude in the lower troposphere (left panels) and propagates into the jet to have peak amplitudes in the upper troposphere at the optimization time of 2 days (right panels).

Yamane and Yoden (2001) also proposed a new efficient method to estimate growing perturbations for a finite-time interval by making a singular value analysis on a subspace spanned by grown Lyapunov vectors with non-negative exponents. The method could be applied for operational NWPs, because the dimension of the subspace is expected to be much smaller than the dimension of the whole system.

f. Some recent progress

Aurell et al. (1997) introduced a finite-size Lyapunov exponent which measures the growth rate of finite-size perturbations and investigated the predictability problem in three-dimensional turbulence with a wide spectrum of temporal scales. They found a universal scaling law in a range where the perturbation is still small compared to large-scale fluctuations, but large compared to the smallest dynamically active scales. The predictability of a flow which possesses many scales of motion was originally investigated by Lorenz (1969) with a simple mathematical model which describes a one-dimensional cascade of error energy in a wide range of wavenumbers derived from a two-dimensional vorticity equation. The scale interactions in predictability experiments were examined by Tribbia and Baumhefner (2004) with an atmospheric general circulation model (GCM) and showed distinctive differences from the classical inverse cascade picture of predictability error growth; the error growth eventually asymptotes to an exponential growth of baroclinically active scales.

The applicability of information theory for predictability studies has been investigated by utilizing some predictability measures based on information-theoretical principles, such as the predictive power (Schneider and Griffies 1999), ignorance (Roulston and Smith 2002), and relative entropy (Abramov et al. 2005). By introducing a measure of sample utility in a relative entropy framework, Haven et al. (2005) estimated uncertainties in predictions coming from relatively small sample size of a forecast ensemble in a non-Gaussian framework.

Smith et al. (1999) investigated the dynamics of an initial uncertainty in the state of a chaotic system during the early states of its evolution. Judd and Smith (2001, 2004) further investigated uncertainty in estimation of the initial state when there is observational error, with the perfect or the imperfect model scenario.

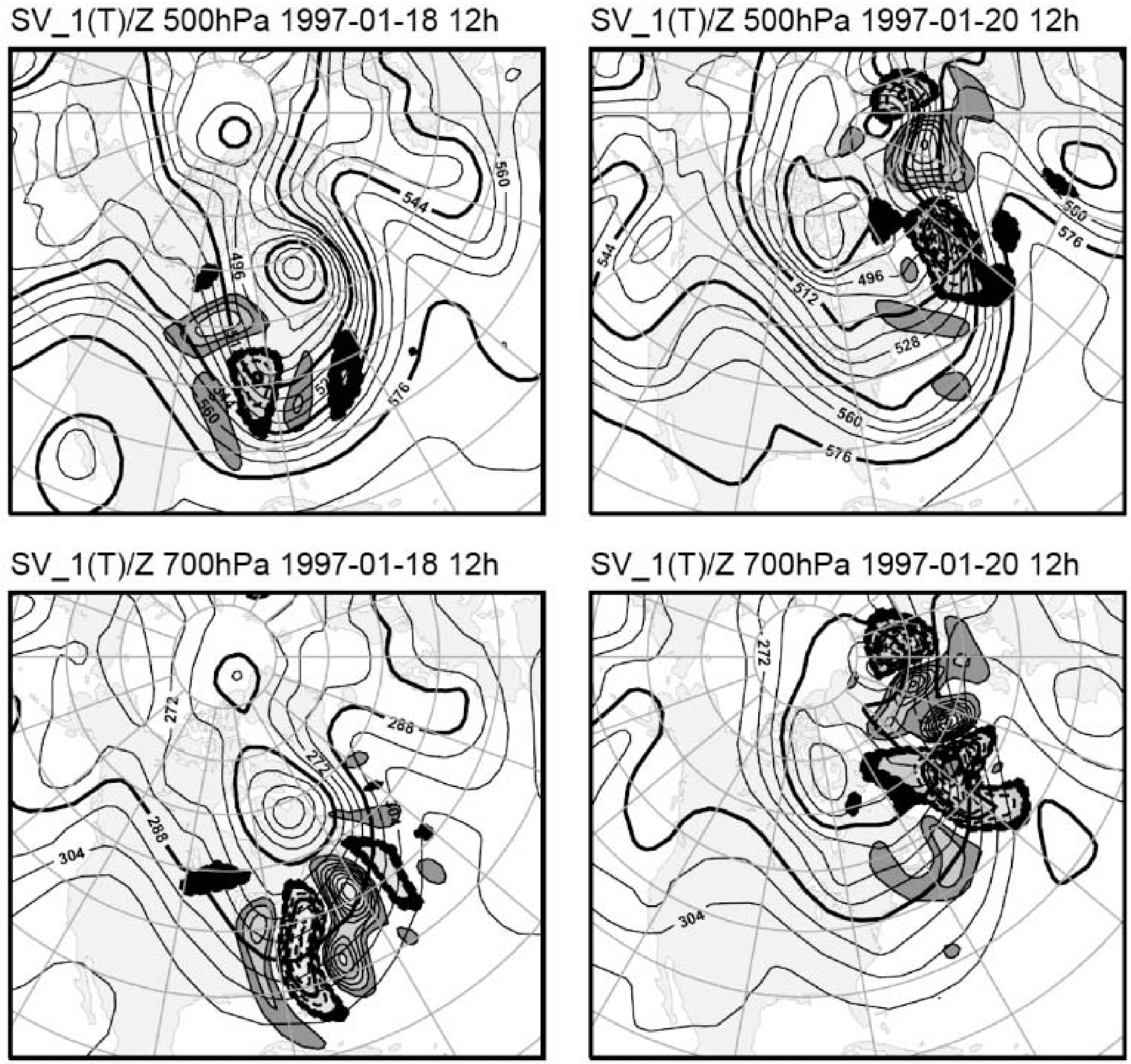


Fig. 8. Singular vector analysis done at ECMWF (Buizza 2001). Most unstable singular vector growing between 18 and 20 January 1997 is shown for temperature component with gray shadings at 500 hPa (top panels) and 700 hPa (bottom panels) for the initial time (left) and final time (right). Contour are for geopotential height [dam] for each pressure level.

They showed the existence of a set of states indistinguishable from the true state and the necessity of probabilistic approach to forecasting the indistinguishable states. They also presented a new method for calculating the maximum likelihood estimate of the true state to perform ensemble forecasts. Judd et al. (2004) applied the shadowing analysis based on these theories of indistinguishable states to an operational NWP system in order to reveal its usefulness to identify the model and analysis errors and to obtain better analyses and forecasts.

3. Ensemble numerical weather predictions

a. From chaos to weather forecasting

The title of the 1991 Kyoto Prize Workshop was “From Weather Forecasting to Chaos” in honor of Lorenz's discovery of chaos in the field of weather forecasting, but the situation in the early 1990's was quite opposite; it was a period characterized by "From Chaos to Weather Forecasting". There were intensive efforts to develop a modern operational system of ensemble NWPs based on the chaos theory.

Finite-time Lyapunov stability stated in the previous section is a tangent linear theory of small errors included in the initial condition. As the errors become finite amplitude, the nonlinear effects become important and the PDF of random errors is not any more an ellipsoid at the stage of $t = t_0 + \tau_{NL}$ as shown in Fig. 7. The time interval during which the linear assumption is valid is dependent on the magnitude of the initial errors and the degree of instability of the system around the initial state. It would be about a day or less in current operational global NWP models. An ensemble NWP is only the plausible way to evaluate the error growth in the nonlinear phase under the current computing facilities. Note that the average of a limited number of ensemble members is not necessarily close to the state corresponding to the maximum of PDF in the nonlinear phase. Such an example of the nonlinear phase will be shown in Section 4.*e*.

b. Generation of initial ensembles based on the chaos theory

In the past when computer resources were limited, some simple methods of ensemble NWPs had been proposed such as the LAF method (Hoffman and Kalnay 1983). The number of ensemble members was very limited and the randomness of the sampling of initial ensembles was not necessarily guaranteed. There was no principle or guideline to determine the initial ensembles efficiently under the limitation of ensemble members for operational NWP models with large degrees of freedom of $O(10^7)$.

Several methods to generate the initial perturbations of ensemble members had been developed at operational centers in medium-range weather forecasts in the early 1990's. One is the *singular vector* (SV) method developed at ECMWF (Mureau et al. 1993; Molteni et al. 1996) and another is the *breeding* method developed at NCEP (Toth and Kalnay 1993, 1997). These methods are based on the chaos theory that describes the growth of small initial perturbations in nonlinear dynamical systems.

In the SV method, a limited number of singular vectors $\boldsymbol{f}_i$ given in Eq. (7) corresponding to the leading singular values are added to or subtracted from the control initial state obtained in the regular analysis by setting the amplitude comparable to the observational errors. The leading singular vectors are computed by an iterative Lanczos procedure using the adjoint model of a simplified version of the forecast model, as the dimension of the models is too large to deal with the error matrix practically. In the breeding method, on the other hand, "bred grown" vectors are used; the bred vectors are the vectors that have been bred for a long time in the analysis cycle, and correspond to the leading $\boldsymbol{\zeta}_i$s in the linear limit.

The advantage of the SV method is that the ensemble members diverge rapidly with time, which seems to be important when the number of ensemble members is limited, while the advantage of the breeding method is that the initial ensemble members tend to be restricted on the attractor of the real atmosphere. Detailed comparison of these methods has been done; for example, Szunyogh et al. (1997) made a direct comparison of SVs and bred vectors in a low-resolution GCM and noted their relationship. However, it remains controversial which method is better or not for generating the initial perturbations of ensemble members. Hamill et al. (2000) also compared these methods and an alternative method of perturbed observation (PO) (Houtekamer and Derome 1995), which approximates a random sample from the analysis PDF by using a Monte Carlo-like observation system simulation experiment to obtain initial perturbations. They showed better performance of the PO method and its new ability to improve data assimilation techniques, by using a quasi-geostrophic channel model coupled with a 3D-variational data assimilation scheme.

c. Operational medium-range ensemble forecasts

ECMWF developed and implemented an ensemble prediction system (EPS) based on the SV method in December 1992, while NCEP introduced a breeding EPS operationally in the

Table 1. Available ensemble prediction systems. This table is a simplified and updated version of a table in the report on the first TIGGE workshop (WMO 2005b) which summarizes some key characteristics of the global ensemble prediction systems run operationally. Abbreviations in the column of Perturbation method are as follows: SV: Singular Vectors, BV: Bred Vectors, EOF: Empirical Orthogonal Functions, EnKF: Ensemble Kalman Filter.

SYSTEM (country)	Runs [/day]	Resolution	Lead Time [hr]	Members [/run]	Pert. method
BMRC (Australia)	2	TL119	240	33	SVs
CMA (China)	1	T106	240	33	SVs+BVs
CPTEC (Brazil)	2	T126	360	15	EOF
ECMWF (EU)	2	TL255	240	51	SVs
FNMOC (USA)	1	T119	240	17	BVs
JMA (Japan)	1	TL159	216	51	BVs
KMA (Korea)	1	T106	192	17	BVs
MSC (Canada)	1	TL149	240	17	EnKF
NCEP (USA)	4	T126–T62	384	11	BVs

same month (Kalnay 2003). The Meteorological Service of Canada (MSC) started operational daily ensemble forecasts in January 1996 with the PO method and replaced it with the ensemble Kalman filter method (EnKF, see Section 4.*d*) in January 2005. Intercomparison of these global EPSs was done by Buizza et al. (2005) for a 3-month period to identify relative strengths and weaknesses of the three systems.

JMA started a one-month ensemble forecasts firstly in March 1996; operational forecasts were done once a week with 10 ensemble members, of which initial perturbations were given by SVs obtained with a simplified linear balance model. As for medium-range (one-week) forecasts, JMA operationally started daily ensemble forecasts based on the breeding method in March 1999. The two EPSs were unified in March 2001; since then one-month ensemble forecasts have been obtained by extended-range runs of one-week forecasts. Since 2003 all the seasonal forecasts of JMA (3-month and 6-month forecasts) have been also done with ensemble forecast techniques. JMA upgraded one-week EPS in March 2006[1], by increasing the number of ensemble members to 51 and the horizontal resolution to TL159 in spectral form (Table 1).

1 http://www.jma.go.jp/jma/jma-eng/jma-center/nwp/nwp-top.htm

Nowadays ensemble forecast is a standard technique in operational centers for medium-range weather forecasts. Some key characteristics of the operational global EPSs are summarized in Table 1 (WMO 2005b). Nine systems are in operation in the world with a lead time from 8 to 16 days and with ensemble members up to 51. Initial ensemble members are given by the SV method or the breeding method in most centers.

d. Multi-parameter-value grand ensembles

Ensemble forecasts from dynamically conditioned initial states as described above are performed assuming implicitly that imperfection of the forecast model has become small enough and expecting that the growth of the initial errors can be reduced by ensemble techniques. However, the small imperfection of forecast models is not always a relevant assumption. Some ensemble techniques can be used to reduce the imperfection of forecast models. For example, uncertainty in the choice of parameter values in physical parameterization schemes has been investigated by perturbing each parameter value from a standard value in multi-parameter-value (or, perturbed physics) ensemble simulations. This ensemble approach can be justified, because many of the parameter values are determined empirically without exact physical principle. A grand ensemble (an ensemble of ensembles) simulation can be done by ensemble runs with initial perturbations for each value of perturbed parameter.

The grand ensemble "prediction" experiment with the largest ensemble members has been done by a unique challenge of climate*prediction*.net[1] for a longer time scale of global warming (Stainforth et al. 2005; Piani et al. 2005). Over 108,000 participants from 188 countries (as of February, 2007) have downloaded an executable version of a full GCM based on the UK Met Office Unified Model to carry out 45 years of simulation on their personal computers and then return their results to the project's servers. They are allocated a particular set of initial conditions and parameter perturbations to run one member of the grand ensemble simulation.

e. Multi-model grand ensembles

Forecast models are not perfect for many reasons; not only the choice of parameter values, but also the choice of physical parameterization schemes, discrete representations of differentials, model domain (model top for global models) and resolutions. Although there is no theoretical way to evaluate uncertainty due to these factors of model imperfection, there have been pragmatic trials of grand ensembles with several forecast models developed in different NWP centers. For example, correlated variations of forecast skills among different centers depending on large-scale flow regimes as shown in Fig. 6 suggest possible usefulness of multi-model ensembles. As part of the PROVOST (PRediction Of climate Variations On Seasonal to interannual Time-scales) project, a multi-model grand ensemble experiment was done with 9-member ensembles for each of four different atmospheric GCMs, providing a promising result, namely, the multi-model ensemble was substantially more skilful than individual-model ensembles (Palmer et al. 2000). The improvement was largely attributed to the increased ensemble size, though better performance of multi-model ensembles is not trivial mathematically.

Similar multi-model grand ensembles for dynamical seasonal prediction were discussed in the report of comparison of seasonal predictions done by five state-of-the-art U.S. modeling groups (Shukla et al. 2000). A multi-model EPS based on seven global coupled ocean-atmosphere models was constructed under the DEMETER[2] (Development of a European Multimodel Ensemble system for seasonal to inTERannual prediction) project and the results indicated that the multi-model ensemble was a viable pragmatic approach to the problem of representing model uncertainty in seasonal-to-interannual prediction (Palmer et al. 2004). Promptness of the data exchange between operational centers is not a severe requirement in seasonal predictions because their lead time is long enough.

For medium-range forecasts, better performance of multi-model grand EPS compared to a single-model EPS has been demonstrated with operational NWP outputs (e.g., Ziehmann 2000; Matsueda et al. 2006). The better performance is likely due to the increased ensemble size, but it is also strongly dependent on the performance of each EPS. An important thing is that such comparison is not straightforward, because there is no unique measure to evaluate the "performance" as a general problem. Moreover, additional "cost" to make multi-model grand ensembles operationally within a limited time interval should be evaluated, such as data transfer among forecast centers, additional computer resources to handle the data, and so on. This is an optimization problem to construct the best performing multi-model grand EPS within an affordable additional cost, and the solution is dependent on the times and the situation of each operational center.

The North American Ensemble Forecast System[3] (NAEFS) is a new multi-model grand EPS run jointly by the MSC, the National Meteorological Service of Mexico, and the U.S. National Weather Service (NWS). A grand ensemble with the global forecast models of MSC and NWS will provide NWP products up to 2 weeks. The NAEFS could be regarded as a pilot activity of TIGGE, the THORPEX Interactive Grand Global Ensemble (Section 4.*e*). In the TIGGE project it is estimated that the total daily data volume around 200 GB will be routinely transferred from different centers around the world to some central data archives with high-speed data transfer technology.

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1 http://www.climateprediction.net/index.php
2 http://www.ecmwf.int/research/demeter/
3 http://www.emc.ncep.noaa.gov/gmb/ens/NAEFS.html

Current operational global EPSs are mostly designed for medium- or longer-range NWPs as stated above, while their products could be used for short-range probability forecasting too if the data transfer is fast enough. In medium-range EPSs the leading components of the initial perturbations are largely associated with baroclinically unstable synoptic disturbances as shown in Fig. 8, so that spread of the ensemble members is too little in the short-range of the predictions. A poor man's EPS, which consists of a set of NWP forecasts from several operational centers using their own analyses and models, samples uncertainties in both the initial conditions and model imperfections (e.g., Atger 1999; Ziehmann 2000). As poor man's EPS is less prone to systematic biases and errors that cause the underdispersive behavior in single-model EPSs, it can be used for short-range probability forecasting of 1 or 2 days (Ebert 2001, 2002). Recently Arribas et al. (2005) investigated the ability of a poor man's EPS for up to 3 days with 14 models from 9 operational centers, and showed that it is an efficient way of producing ensemble forecasts in the short range, although the ensemble size is still limited.

f. Ensemble forecasts with mesoscale limited-area models

The predictability problem associated with mesoscale phenomena in short-range forecasting is largely different from that of global medium-range forecasting, because of the important roles of dry and moist convective disturbances which grow much faster than large-scale baroclinic disturbances (e.g., Kalnay 2003). Another factor inherent in mesoscale forecasting is the lateral boundary conditions, which might become a basic limitation to predictability with a limited-area model (e.g., Warner et al. 1997).

In the early 1980s, there were some basic researches to apply predictability concepts, which had been developing in those days, to the modeling and prediction of mesoscale phenomena (e.g., Anthes 1984; Warner et al. 1984; Anthes et al. 1985). However, limited-area models were also in developing stage in those days, so that early findings were largely influenced by the poor performance of such models. About a decade later, finite-time Lyapunov stability analysis (or, SV analysis) was firstly applied for a limited-area, mesoscale primitive-equation model by Ehrendorfer and Errico (1995) with the dry-adiabatic version of the National Center for Atmospheric Research (NCAR) Mesoscale Adjoint Modeling System. They investigated domain-internal tangent-linear error growth and showed that only small fraction (0.25%) of SVs grows for a 24-hour time interval, and cleared up some confusion on mesoscale predictability in early days such that the mesoscale was inherently more predictable than the larger scales due to strong constraint by topography and other surface features. SVs in the same system with moist physics were investigated and much faster growth of perturbations with new structures when compared to dry situation was pointed out by Ehrendorfer et al. (1999) and Errico and Raeder (1999).

The importance of moist processes in domain-internal nonlinear error-growth has been further studied by e.g., Zhang et al. (2002, 2003), Errico et al. (2004), Walser et al. (2004), and Hohenegger et al. (2006), in some typical situations with high-resolution nonhydrostatic cloud-resolving models. Hohenegger et al. (2006) revealed significant loss of predictability occurs over moist convectively unstable regions that are able to sustain propagation of energy against the mean flow. All of these studies are indicative that both of moisture analysis and consideration of initial moisture uncertainty are very important in operational short-range ensemble NWPs.

Real-time multi-model ensemble forecasts were performed under the Storm and Mesoscale Ensemble Experiment (SAMEX) during May 1998 over the continental U.S. and central-southern Great Plains (Hou et al. 2001). A multi-model grand ensemble of 25 members with four different mesoscale models showed much better performance than each individual ensemble system, and the reason was considered to be its more realistic representation of the uncertainties in both models and initial conditions. Currently the Environmental Modeling Center of NCEP is running the Short-Range Ensemble Forecasting[1] (SREF) system

1 http://www.emc.ncep.noaa.gov/mmb/SREF/SREF.html

that consists of three models with some different convective schemes with 21 grand ensemble members. A number of university groups in the U.S. have also attempted mesoscale SREF focusing on a limited area. For example, 72-hour forecasts of U.S. Pacific Northwest weather are produced twice per day at the University of Washington[1] (Grimit and Mass 2002). This is a multi-analysis ensemble system; initial conditions and lateral boundary conditions for a single mesoscale model ensemble are routinely obtained from eight operational weather prediction centers worldwide. In Europe, several limited-area ensemble prediction systems are run operationally or in research mode (WMO 2005b); Consortium for small-scale modeling (COSMO) is such an operational system with the non-hydrostatic limited-area model, Lokal Modell (Marsigli et al. 2005).

4. THORPEX challenges

a. THORPEX (THe Observing system Research and Predictability EXperiment)

THORPEX[2] is a ten-year international research and development program with the implementation phase from 2005 to 2014 in order to accelerate improvements in the accuracy of one-day to two-week high-impact weather forecasts for the benefit of society, the economy, and the environment (Shapiro and Thorpe 2004; Rogers et al. 2005; WMO 2005a). The high-impact weather forecasts are typically associated with tropical and extratropical cyclones with mesoscale disturbances which cause gusty winds, heavy rainfalls, blizzard snows, dust-storms, and so on. They also encompass persistent or slowly varying meteorological conditions that affect heat wave, cold wave, and drought.

What THORPEX will do is summarized as an end-to-end forecast system as illustrated in Fig. 9 (WMO 2005a). The combination of basic and applied research on fundamental issues of THORPEX is now conducted to develop and test new observational techniques and systems including targeted observations, new data assimilation and prediction systems, and new decision support systems for social, economic, and environmental decisions. Some highlights in each item and related topics are subjectively selected and briefly described in the following subsections.

b. New observational techniques

Any weather predictions start from observations of current atmospheric states. Nowadays large part of meteorological data used in operational NWPs are obtained by remote sensors installed on geostationary or polar-orbiting satellites. Another type of remote sensors are ground-based precipitation radars and wind profilers. Traditional measurements at surface stations over lands and those on ships and buoys over oceans are important to provide ground truth and long-term records for climate monitoring. *In-situ* upper-air soundings by radiosondes and commercial aircrafts give vertical profiles of some physical quantities necessary for NWPs.

During the time frame of THORPEX, current satellite-based imagers and sounders are expected to have much higher spatial, temporal, and spectral-band resolutions due to the advancement of sensor technology. New satellite-based observation technologies developed and tested in this decade will be further improved and utilized for real-time operational NWPs; such new technologies include active microwave sensors (e.g., NASA WINDS[3] for surface wind-vector measurement over the global oceans and NASA/JAXA TRMM[4] for tropical rainfall measurement) and GPS-LEO occultation soundings for vertical profiles of temperature and water vapor over the globe (e.g., Taiwan-U.S. COSMIC[5]).

Surface observations and ground-based upper-air soundings with either *in-situ* or remote sensors have also been improved drastically over the last decades by advanced sensing and communicating technologies and will complement and enhance the utility of satellite measurements. Some new deployments of ground-based upper-air sounding systems (e.g., radiosondes and rocketsondes), dropsonde systems (e.g., aircraft and stratospheric balloons), unmanned airborne vehicles, high-performance balloons, and so on will become key elements for interactive forecasting and targeted observations.

1 http://www.atmos.washington.edu/~ens/uwme.cgi
2 http://www.wmo.int/thorpex/
3 http://winds.jpl.nasa.gov/index.cfm
4 http://trmm.gsfc.nasa.gov/
5 http://www.cosmic.ucar.edu/index.html

c. Targeted observations

The concept of targeted (or, more generally, adaptive) observing strategy had been developed in association with the implementation of predictability theories into operational medium-range ensemble forecasts (Snyder 1996). Sensitivity studies based on tangent linear analyses in operational NWPs have shown that the rapid growth of initial errors is generally localized in relatively small regions as shown in Fig. 8 and the sensitive regions vary in location day by day, depending on the character of the large-scale flow field. If the observational error in the sensitive regions is reduced by taking additional observations as stated in the previous subsection adaptively within a short enough time interval, improved forecast skill can be obtained at minimal cost. Some theoretical arguments on targeted observations based on singular and bred vectors can be found in e.g., Palmer et al. (1998), Lorenz and Emanuel (1998), Bishop and Toth (1999), and Buizza and Montani (1999).

The practical feasibility and potential impact of targeted observations was tested in the field phase of the FASTEX (Fronts and Atlantic Storm-Track EXperiment) project in January and February of 1997 (Joly et al. 1999). It was the first real-time adaptation of the observations to areas critical to improving predictions for generation and growth of extratropical cyclones. The impact of the targeted observations was assessed by using the operational models of NCEP (Szunyogh et al. 1999; Pu and Kalnay 1999), ECMWF (Montani et al. 1999), NOGAPS (Navy Operational Global Atmospheric Prediction System; Langland et al. 1999; Gelaro et al. 1999), and ARPEGE/IFS (Action de Recherche Petite Echelle Grande Echelle/Integrated Forecasting System; Bergot 1999). Montani et al. (1999) reported the reduction of short-range (up to day 2) prediction errors with a maximum of 37% inside the SV verification regions, though there were some cases when the extra data degraded the forecasts (e.g., Szunyogh et al. 1999).

Some targeted observations in association with TIGGE have been and will be performed for improving high-impact weather forecasts as THORPEX Regional Campaigns (TReCs) in Asia, Europe, North-America, Southern-Hemisphere, and combinations of these regions. A North Atlantic THORPEX Regional Campaign (NA-TReC) was already conducted from mid-October to mid-December 2003, including special observations with dropsondes and radiosondes, and extra data from geostationary satellites and commercial aircraft. The observation impact was evaluated by Langland (2005) using an adjoint-based diagnostic technique.

d. New data assimilation techniques

Data assimilation is a contemporary method to estimate the initial condition (or, "analysis" of the current state) for NWP from the current and past observed data. For details on data assimilation, see the review article by Tsuyuki and Miyoshi (2007) in this special issue and the textbooks by e.g., Daley (1991), Kalnay (2003), and Evensen (2007). It is an important component to reduce uncertainty in the initial conditions, and some statistical methods such as optimum interpolation had been used to obtain the "best" analysis in the past. In the last decades, the data assimilation schemes have been improved very much by their incorporation in NWP systems under a new discipline where dynamics is merged with observations, such as four-dimensional variational data assimilation schemes (4D-Var). It is superior in handling asynchronous observational data that are taken at any time or place rather continuously by e.g., instruments on satellites and aircrafts. Merging of dynamics and observations could have multiple meanings in the NWP systems with 4D-Var, because the adjoint model developed for 4D-Var can be used to generate initial perturbations of ensemble forecasts with the SV method. However, it is necessary to maintain the adjoint model as the evolution of the corresponding NWP model, and large efforts are generally required for the maintenance of the highly complicated computational codes.

Ensemble Kalman filter (EnKF) is a new stream of assimilation method which unifies ensemble forecasts and data assimilation. It was first introduced to a quasi-geostrophic system by Evensen (1994) and several innovations have been done to improve the accuracy and reduce the computational cost, such as square root filters (e.g., Anderson 2001), local ensemble Kalman filter (LEKF) method (Ott et al. 2004), and local ensemble transform Kalman

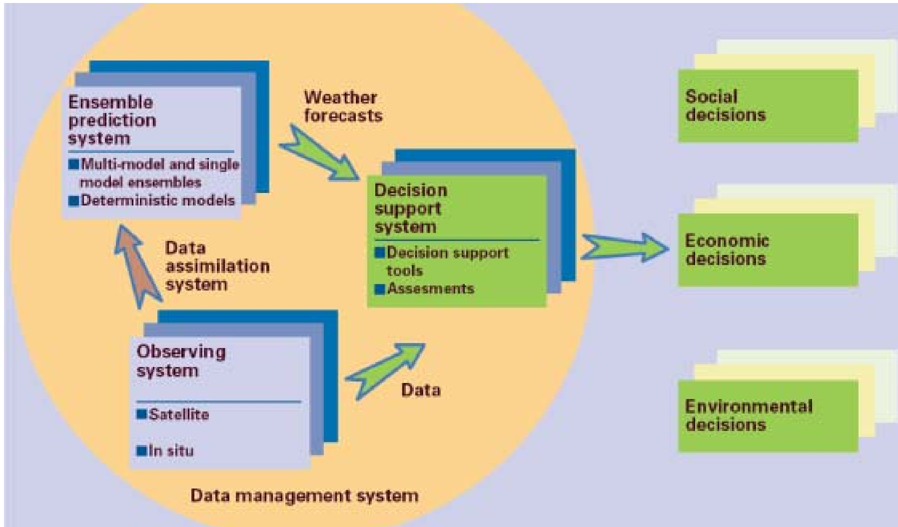

Fig. 9. End-to-end forecast system as a future global interactive forecast system (WMO 2005a).

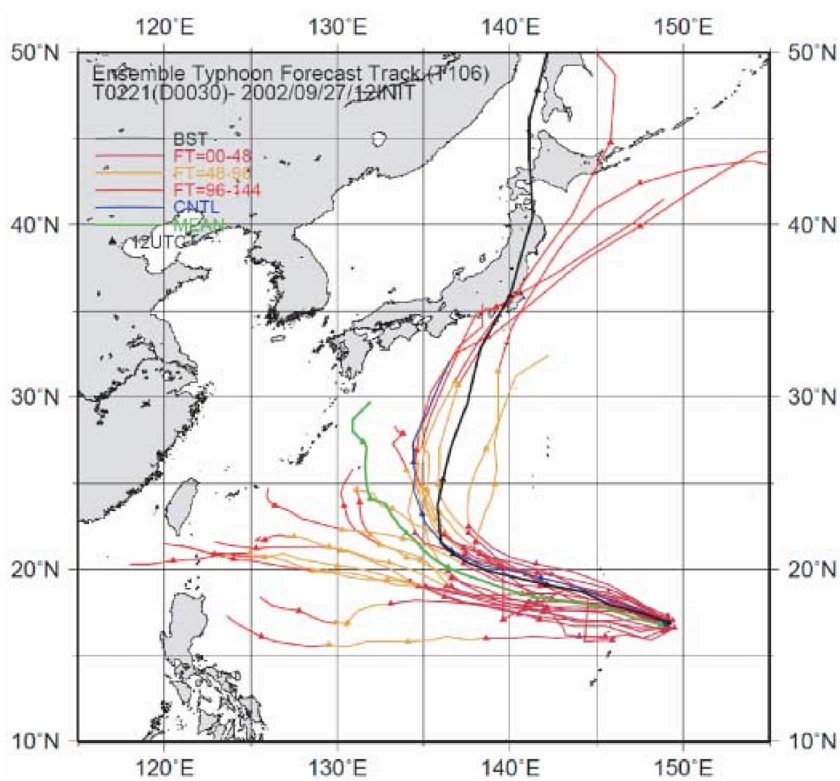

Fig. 10. Ensemble forecasts of typhoon track of HIGOS (T0221) in late September, 2002 (Sakai and Yamaguchi 2006). Red and orange lines are the results of 144-hour ensemble forecasts from September 27 12UTC 2002 with triangles at every 12 hours, and thick green line denotes the ensemble average. Thick black line is the observed best track (BST).

filter (LETKF) method (Hunt 2006). Recently developed four-dimensional LETKF was applied to the atmospheric GCM for the Earth Simulator for the test of computational efficiency and parameter sensitivity in full global GCM (Miyoshi and Yamane 2006) and to the JMA Nonhydrostatic Model for the test of convective-scale data assimilation and ensemble prediction (Miyoshi and Aranami 2006). In these studies encouraging results were obtained for the new method as possible operational systems in the future. The next 5–10 years will show whether EnKF becomes the operational approach of choice, or 4D-Var and its improvements remains the preferred advanced data assimilation method (Kalnay et al. 2006).

Another challenge in data assimilation is "adaptive data assimilation". Drastic increase of data amount is expected by high-resolution satellite-based observations in near future as stated in Section 4.*b*. Effective thinning of large datasets to retain the most useful observations is an important subject to obtain the best estimation of initial conditions. The same strategy as target observation can be taken for this data thinning in operational NWPs. The information of time-dependent sensitive regions obtained in generating initial perturbations for ensemble forecasting will be utilized to retain high-density data only in the sensitive regions in the assimilation process.

e. New prediction systems

THORPEX will accelerate improvements of the accuracy of one-day to two-week weather forecasts by testing and demonstrating effectiveness of new prediction systems. This includes the TIGGE project that integrates developments in observing systems, targeted observations, adaptive data assimilation, model improvements, forecast user requirements, and a multi-model/multi-analysis EPS (Shapiro and Thorpe 2004; WMO 2005b). Ensemble techniques stated in Section 3 become key components of TIGGE. One of the challenges of TIGGE is a feasibility study of an *interactive* EPS which responds dynamically to changing uncertainty to forecast errors, including the use of adaptive observations, variable ensemble size, and on-demand regional ensembles.

Typhoon EPS recently developed at JMA is an example of on-demand ensembles for the regional purpose of typhoon track prediction. A newly developed SV method with full physical processes was incorporated into a global ensemble NWP system with a horizontal resolution of about 60 km (Sakai and Yamaguchi 2006). Figure 10 shows an example of ensemble forecasts of typhoon track of HIGOS (T0221) in late September 2002. A cluster of ensemble members predicts the track very well, while another cluster shows straight track westward. In this particular case, the ensemble mean denoted by the thick green line is not the best way to diagnose the typhoon track. Methods for efficient utilization of ensemble prediction data are under investigation, such as a strike probability map, time-series plots at a given place, and so on. Operational use of the JMA typhoon EPS will be started in the near future.

As shown in Fig. 10, ensemble forecasts of typhoon track occasionally show large uncertainty beyond some limited area; in this case the tracks diverge around 20 N and 135 E. If we can determine the sensitive region for the forecast around this limited area by the SV method in a short time interval, some targeted observations may be deployed to reduce the forecast uncertainty of the typhoon track, such as super rapid scan operations[1] of geostationary satellite, dropsondes, unmanned airborne vehicles, or something else. Such targeted observations and refinement of the ensemble forecasts of typhoons in the Western Pacific are under planning as an Asian TReC for the 2008 typhoon season.

f. Decision support systems

Interaction between the makers of weather forecasts and their users has been rather limited owing to the gap in communication. In order to enhance the utility and value of weather forecasts to society, economies, and environmental stewardship, THORPEX will develop and apply new decision support systems through user-specific probabilistic forecast products as illustrated in Fig. 9 (WMO 2005a). Ensemble forecasts have greater potential economic value than corresponding single deterministic forecasts with uncertain accuracy, by

1 http://www.data.kishou.go.jp/satellite/rapid.html (in Japanese)

using probabilistic information as input to decision-model analyses (Palmer 2002a).

Palmer (2002b) gave some examples of the commercial application of ensemble forecasting, such as electricity generation, ship routeing, pollution modeling, weather-risk finance, crop yield modeling, and disease prediction. Recently, an early warning system of malaria in southern Africa based on seasonal forecasts from DEMETER was developed, relying on the relationship between climate variation and malaria incidence through both mosquito vector dynamics and parasite development rates (Thomson et al. 2006). The system was successfully applied to the prediction of malaria risk in Botswana with four months lead time. Taylor and Buizza (2006) used ECMWF ensemble predictions with 10-day lead time to forecast the density of the payoff from a weather derivative. It is a real application of medium-range ensemble forecasts in the financial sectors.

In order to develop such a system to forecast probabilities of some user-specific measure with an ensemble NWP system, *re-forecast* (i.e., retrospective forecasts) dataset over a long period is necessary to obtain any relationship translating predicted meteorological parameters at specific spatial and temporal scales into societal and economic attributes of the natural or human environment (Shapiro and Thorpe 2004). A measure based on predicted precipitation, temperature, wind speed, or else should be related to a specified attribute of energy demand, agricultural production, transportation efficiency, demands on health services, management of water resources, decision making in commodity markets, or else. Ensemble reforecasts is a very important dataset not only for improving weather predictions (Hamill et al. 2006) but for developing decision support tools for social, economic, and environmental decisions.

5. Concluding remarks

The success of numerical weather prediction represents one of the most significant scientific, technological, and societal achievements of the 20th century as stated by Shapiro and Thorpe (2004). It was directly or indirectly related to the progress in dynamic meteorology and nonlinear mathematical sciences and to the advancement of computer science and technology, and it will remain one of the major challenges of simulation science with high-performance computing systems.

Atmospheric predictability research based on chaos theories has realized operational ensemble prediction systems, which are highly sophisticated computing systems. Under the THORPEX project some further challenges in predictability research will be taken; development of interactive grand global ensemble prediction systems deploying some targeted observations, unification of ensemble forecasts and data assimilation with ensemble Kalman filter technique, development of new decision support systems within an end-to-end forecast system, and so on. The predictability research will be accelerated through international collaboration among academic institutions, operational forecast centers, and users of forecast products (WMO 2005a).

It is my pleasure if this review article could make some contribution to getting perspectives on atmospheric predictability research in the 2nd century of numerical weather predictions.

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Appendix

List of acronyms

ARPEGE/IFS	Action de recherche petite echelle grande echelle/Integrated forecasting system
COSMIC	Constellation observing system for meteorology, ionosphere, and climate
COSMO	Consortium for small-scale modeling
DEMETER	Development of a European

	multimodel ensemble system for seasonal to interannual prediction
ECMWF	European Centre for Medium-Range Weather Forecasts
ENIAC	Electronic Numerical Integrator And Computer
EnKF	ensemble Kalman filter
EPS	ensemble prediction system
FASTEX	Fronts and Atlantic storm-track experiment
FLOPS	floating point number operations per second
GCM	general circulation model
GPS	global positioning system
IAS	Institute for Advanced Study
JAXA	Japan Aerospace Exploration Agency
JMA	Japan Meteorological Agency
LAF	lagged average forecast
LEKF	local ensemble Kalman filter
LETKF	local ensemble transform Kalman filter
LEO	low earth orbit
MEXT	Ministry of Education, Culture, Sports, Science, and Technology
MSC	Meteorological Service of Canada
NAEFS	North American ensemble forecast system
NASA	National Aeronautics and Space Administration
NCAR	National Center for Atmospheric Research
NCEP	National Centers for Environmental Prediction
NMC	National Meteorological Center
NOGAPS	Navy operational global atmospheric prediction system
NWP	numerical weather prediction
NWS	National Weather Service
ODE	ordinary differential equation
PDF	probability density function
PO	perturbed observation
PROVOST	Prediction of climate variations on seasonal to interannual time-scales
SAMEX	Storm and mesoscale ensemble experiment
SREF	short-range ensemble forecasting
SV	singular vector
THORPEX	The observing system research and predictability experiment
TIGGE	THORPEX interactive grand global ensemble
TReCs	THORPEX regional campaigns
TRMM	Tropical rainfall measuring mission
WMO	World Meteorological Organization
4D-Var	four-dimensional variational data assimilation scheme

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